# Senior 4 Pre-Calculus Mathematics (40S) Standards Test

# Administration Manual for Supervising Teachers

June 2003

Manitoba

Building for the Future

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Manitoba Education and Youth,
Assessment and Evaluation Branch,
Winnipeg, Manitoba R3E 3J5

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## Preface

This *Administration Manual for Supervising Teachers* is distributed to teachers and administrators, and must be read prior to administering the standards test. The rules and guidelines in this manual must be strictly observed to ensure the consistency of standards test administration.

Please use this manual in conjunction with the registration package, the *Policies and Procedures for Standards Tests* handbook and the *Information Bulletin* to gain a full understanding of the protocols and procedures associated with this test. This manual contains general information about the test, step-by-step instructions for the day of the test administration, and policies regarding absenteeism, lateness, violation of rules, etc.

If you encounter exceptional circumstances not addressed in this manual, or if you have further inquiries, contact Carol Chabbert, Administrative Officer, at 945-0747, or toll-free at 1 800 282-8069, extension 0747.

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# General Information

Manitoba Education and Youth develops all standards tests in collaboration with teachers from across the province. This standards test is based on the Senior 4 Mathematics Manitoba Curriculum Framework of Outcomes and Senior 4 Standards (2000).

The standards test is locally marked and counts for 30% of each student's final grade in the course. Marking instructions are provided by the department immediately following test administration. School principals will receive Senior 4 Pre-Calculus Mathematics (40S) Standards Test *Marking Guides* by June 10, 2003. The *Marking Guides* are to be stored in a secure location until 12:30 p.m. on June 10, 2003. Student results on the test will be released to students according to local policy on the date designated by local jurisdictions.

All students enrolled in Senior 4 Pre-Calculus Mathematics (40S), except those who have been exempted, must write the *Senior 4 Pre-Calculus Mathematics (40S) Standards Test* on June 10, 2003. However, a student may rewrite a mandatory provincial standards test at any regularly scheduled administration without being registered in the course (see *Policies and Procedures* for Standards Tests, Section 3.1)

Students are expected to write the standards test as scheduled near the end of the semester in which they take the course (e.g., January for first-semester courses, May/June for second-semester and full-year courses).

All schools should receive the test materials by June 2, 2003. Principals of public schools receive test materials from their division/district offices. Principals of independent and band-operated schools will receive test materials directly from the Assessment and Evaluation Branch.

The number of test booklets sent to each school is based on the February 21, 2003 registration information provided by the school. Please advise Carol Chabbert, Administrative Officer, at 945-0747, or toll-free at 1 800 282-8069, extension 0747, immediately if the number of students registered to write the test has increased since registration.

In the context of marking standards tests at the local level, Manitoba Education and Youth will:

- train representatives from local jurisdictions to act as local marking co-ordinators;
- collect and scan all *Answer/Scoring Sheets* from jurisdictions to provide them with test result summaries;
- centrally mark a random sample of locally-marked test booklets to provide feedback to jurisdictions on their local marking; and
- support provincially-funded local jurisdictions by providing $10.00 for each locally-marked test administered in accordance with the test schedule noted on the following page.

## Features of the test

The *Senior 4 Pre-Calculus Mathematics (40S) Standards Test* consists of two parts:

- Part 1—open-response questions where a scientific or graphing calculator may be used; and
- Part 2—selected-response, restricted-response and open-response questions where a calculator may not be used.

## Test schedule

The following table summarizes the schedule for the *Senior 4 Pre-Calculus Mathematics (40S) Standards Test*:

| Date | Session | Preparation Time | Activity | Time Required for Test |
|---|---|---|---|---|
| Tuesday, June 10, 2003 | 1 | 15 minutes preparation and instructions | Part 1 | 60 minutes |
| | | | Part 2 | 120 minutes |

**The test must be written between 8:30 a.m. and 12:30 p.m. on Tuesday, June 10, 2003.**

# Policies for Standards Tests

Principals and supervising teachers should be aware that *Policies and Procedures for Standards Tests* handbook has been distributed to all schools and is available on-line at http://www.edu.gov.mb.ca/ks4/assess/publications.html. This handbook outlines current policies and procedures related to the administration of standards tests and reflects the principles and guidelines contained in *Principles for Fair Student Assessment Practices for Education in Canada (1993)*.

## Adaptations

Schools are asked to identify and submit requests for test adaptations **at the time of student registration**. These requests should be submitted on the *Request for Adaptations Form* included with the registration package. Following departmental review of the adaptation requests received, schools will be advised of the department's decision regarding these requests (see *Policies and Procedures for Standards Tests*, Sections 4.0, 4.1 and 4.2).

## Exemptions

Decisions for student exemptions that fall within the guidelines set out in *Policies and Procedures for Standards Tests* (Sections 5.0, 5.1 and 5.2) are made at the school level and do not require departmental approval. Schools are required, however, to report school-based exemptions on the *Student Exemption Report* included with the registration package. If additional student exemptions are needed after the registration deadline date, an additional *Student Exemption Report* should be submitted as soon as possible.

For exemptions that fall outside the parameters of policy and procedure guidelines, requests should be made **at the time of student registration**. Please submit requests in writing to Ernest Molgat, Director, Assessment and Evaluation Branch, 71–1567 Dublin Avenue, Winnipeg MB R3E 3J5.

## Re-read provisions

As the reporting of individual student scores on locally marked standards tests is the responsibility of each local jurisdiction, the re-read policy for these tests is determined locally. Should a re-read be required for a test booklet that has been sent to the department as a part of the feedback marking process, the department will return a photocopy of the test booklet upon request. Please contact Carol Chabbert, Administrative Officer, at 945-0747, or toll-free at 1 800 282-8069, extension 0747.

## Return of tests to students

Local jurisdictions decide whether to keep the test booklets or return them to students. The department will return the random sample of test booklets to schools within three months of marking.

## Test confidentiality

Test materials must be kept secure and sealed until administration.

Test booklets must not be reproduced nor information from them communicated in any way until test administration is completed.

The *Marking Guides* are to be stored in a secure location until the test administration is complete.

## Student materials

Students may use the following materials as specified, during the *Senior 4 Pre-Calculus Mathematics (40S) Standards Test*:

- HB lead pencil
- calculator (scientific or graphing—Part 1 only)
- eraser
- ruler

Students are not permitted to exchange materials during the test.

The following materials must **not** be used during the test:

- class notes
- notebooks
- textbooks
- dictionaries

## Students leaving the room

Students are expected to remain in the room for Part 1 (60 minutes) and until they have completed Part 2 of the test.

However, any student who must leave and re-enter the room should be accompanied by a replacement supervisor. This practice ensures that the supervising teacher can remain in the test room with the students at all times.

## Late arrivals

Students who arrive more than one hour after the test has started are not allowed to write the test. Students who arrive late but within the first hour of the test are permitted to write but no extra time is allowed. The exception will be if students arrive late for reasons beyond their control. Under these circumstances, the local jurisdiction may allow extra time to write the test. The test however, may not start after 9:30 a.m. Please note that late students who are granted extra time may only use their calculators for Part 1 of the test (maximum of 60 minutes). The classroom teacher should record on the *Supervising Teacher's Report* the booklet numbers of students who arrived late and provide details of the circumstances which caused the student(s) to arrive late.

## Student absences

Standards tests cannot be written at a later date and reported as a standards test result. Therefore, schools have the responsibility to provide a final mark to absent students. The final mark for these students must be identified as a school-based assessment and evaluation practice only (see *Policies and Procedures for Standards Tests*, Sections 6.0, 6.1 and 6.2 for additional information).

## Responding to student questions

Avoid making any directing/guiding comments during the test, other than those required for distribution of materials and instructions to students.

No explanations, translations, rephrasing, clarifying statements or comments of a similar nature may be provided about the test content unless otherwise specified. Classroom teachers can, however, provide clarification regarding test-taking rules or procedures.

If a student asks a question related to test content, a suggested response is, "I'm sorry. I can't answer your question. Do your best with it."

Suspected errors in printing or wording should not be communicated to students unless specific instructions are received from the Assessment and Evaluation Branch. You may however, contact Neil Vadeboncoeur, Assessment Consultant, at 945-6715 or Chantal Bérard, Assessment Consultant, at 945-7590 or toll-free at 1 800 282-8069, extensions 6715 or 7590 respectively to report any suspected errors.

## Violation of rules

The *Test-Taking Rules* are read aloud to students by the supervising teacher immediately before the test (see Appendix 1, p. 17).

Document any violation of rules on the *Supervising Teacher Report*. This documentation should include:

- the test booklet number; and
- a description of the circumstances, including available evidence.

The student may be permitted to complete the test.

The consequences of any violation of rules is a local decision.

## Damaged or misprinted material

Students should be assured that they will not be penalized for any obvious error in printing or wording. Local marking co-ordinators will be advised of any known errors before the marking session. Return the unused damaged booklet(s) with the reports and forms being sent to the department immediately after the test session.

### On test administration date

If a damaged or misprinted test booklet is discovered **at the start** of the test session:

- replace the damaged or misprinted test booklet with a new test booklet; and
- print "DAMAGED" on the test booklet cover and on the *Attendance Form*.

### During test administration

If a damaged or misprinted test booklet is discovered **during** the writing of the test:

- give the student a new test booklet (the student keeps the damaged test booklet until the end of the session);
- instruct the student to continue responding to test items in the new test booklet;
- collect both test booklets at the end of the test session;
- cross out the existing booklet number on the undamaged test booklet and change it to the original booklet number so that **all student responses are recorded under the original booklet number**;
- indicate on the *Attendance Form* that two booklets have been used for one student (see Appendix 3, p. 19);
- record the damaged or misprinted test booklet number on the *Supervising Teacher Report*; and
- ensure both booklets are kept together by bundling with an elastic band.

# Responsibilities of School Principals

It is the responsibility of the principals to ensure that the tests are kept confidential until test administration is completed.

Other responsibilities are listed below.

If tests are not in your school by June 2, 2003:

- public school principals should contact their superintendent;
- principals of independent and band-operated schools should contact Carol Chabbert, Administrative Officer, at 945-0747, or toll-free at 1 800 282-8069, extension 0747.

Use the *Packing List* to confirm that all materials have been received. Count the shrink-wrapped test booklets to ensure that there are enough booklets for the number of students writing the test. Keep the shrink-wrapped test packages unopened until the morning of the test. If additional test materials are needed, immediately contact Carol Chabbert, Administrative Officer, at the number noted above.

Store test materials in a secure area in the school.

Provide supervising teachers with a listing of the Manitoba Education and Youth (MET) numbers for the students registered to write the test. Call the Professional Certification and Student Records Unit at 1 800 667-2378 to obtain MET numbers for new students.

Make arrangements to meet the requirements of the adaptations approved for your students (see Adaptations, page 3 and *Policies and Procedures for Standards Tests*, Sections 4.0, 4.1 and 4.2).

Co-ordinate test scheduling and space requirements.

Minimize the frequency of school announcements during the test session.

Ensure that a replacement supervisor is available to accompany students who must temporarily leave the room.

Provide the test materials to the supervising teacher on the morning of the test.

Complete the *Principal Feedback Form* before the end of the test session.

Advise the Assessment and Evaluation Branch of school closures or suspended school bus transportation services that occur on the test date(s) as a result of unusual or extenuating circumstances or inclement weather conditions. Please contact Neil Vadeboncoeur, Assessment Consultant, at 945-6715 or Chantal Bérard, Assessment Consultant, at 945-7590 or toll-free at 1 800 282-8069, extensions 6715 or 7590 respectively.

# Responsibilities of Supervising Teachers

Supervising teachers administer the test using the step-by-step procedures outlined in the following section ("Checklist of Procedures") in this manual.

## Points to Remember

- Remain in the classroom throughout the entire test session and engage in work solely related to the duties of supervising the test.
- A replacement supervisor must be available to assist you.
- Store all documents not in use in a secure area in the school.
- Ensure that once a booklet number has been assigned to a student, all materials distributed to that student have the same booklet number.
- Students **may** review or complete work from Part 1 during the Part 2 portion of the test session. Students **may not**, however, use their calculator during Part 2 of the session.
- Students **may not** bring anything produced outside of class into the test session.

## Checklist of Procedures

| | |
|---|---|
| | **Advance Preparations** |
| | 1. Check with your principal to ensure that: |
| | • appropriate scheduling and space allocation for the test has occurred. |
| | • the test materials itemized on the *Packing List* have been inventoried. |
| | • a replacement supervisor is available to accompany students who must temporarily leave the room during the test. |
| | • arrangements have been made to accommodate the adaptations approved for students. |
| | • a list of MET numbers for students writing the test is available. |
| | 2. Ensure that all subject-related materials have been removed from the walls and boards. |
| | 3. Notify students in advance of the: |
| | • time, location and duration of the test session. |
| | • materials needed to write the test. |
| | • *Test-Taking Rules* (see Appendix 1, p. 17). |
| | • requirement to remain in the test session for the allotted time for Part 1 (60 minutes) and until completion of Part 2. |
| | 4. Ensure that you have extra supplies and that there is a functioning pencil sharpener and stapler in the room. Students must not exchange materials during the test. |
| | 5. If there is more than one teacher supervising the test, photocopy the *Supervising Teacher Report*. |
| | 6. Review the sample forms in the Appendices. |
| | **Prior to test session** |
| | 1. Ensure that the following materials are available: |
| | • *Packing List*; |
| | • *Senior 4 Pre-Calculus Mathematics (40S) Standards Test* booklets—Part 1 and Part 2; |
| | • *Attendance Forms* (enclosed with shrink-wrapped test booklets); |
| | • *Student Registration Update Form*; |
| | • extra copies of test booklets (four extras); |
| | • *Demographic Data Forms*; |
| | • *Answer/Scoring Sheets*; |
| | • *Supervising Teacher Report*; |
| | • *Supervising Teacher Feedback Form*; |
| | • *Principal Feedback Form* (this must be given to the principal and collected before returning materials); |
| | • *Administration Manual for Supervising Teachers* (this document); |
| | • Xpresspost Flexipack envelope (for schools located outside the city of Winnipeg). |

| | |
|---|---|
| | 2. Count the number of *Senior 4 Pre-Calculus Mathematics (40S) Standards Test* booklets, Part 1 and 2. The number of test booklets should equal the number requested by the school plus the additional booklets. The number of booklets should also be identical to the number on the *Packing List*.<br>If you need to split a bundle of booklets between classrooms, make a photocopy of the *Attendance Form* enclosed with the bundle. **On your copy**, cross out the test booklet numbers that are not used in your classroom and forward these booklets to the supervising teacher who requires them. |
| | 3. Arrange the desks so they are spaced as far apart as possible. |
| | 4. Ensure that students exempted from writing the test are not present. |
| | **Beginning the test session** |
| | 1. Distribute: |
| | • *Senior 4 Pre-Calculus Mathematics (40S) Standards Test* – Part 1. Part 2 is distributed after 60 minutes; |
| | • *Demographic Data Forms*. |
| | 2. Tell students that while they are working you will be circulating the *Attendance Form* so they can print their names beside their test booklet number. |
| | 3. Read instructions for completing *Demographic Data Forms* printed on the top left corner of the form. Direct students to print and shade in all the information requested on the *Demographic Data Form*. Remind students of the importance of accurately recording the program (English, Immersion or Français) they are now in. |
| | 4. Collect the *Demographic Data Forms*. |
| | 5. Read aloud the *Test-Taking Rules* to students (Appendix 1, p. 17). |
| | 6. Read aloud *General Directions* to students (inside the front cover of Part 1 test booklet). |
| | 7. Remind students to: |
| | • pace themselves in writing the test. |
| | • use the *Answer/Scoring Sheet* which will be distributed with Part 2 of the test to record their answers to the selected-response questions. |
| | • check over their work to ensure that all questions have been attempted. |
| | 8. **Instruct students to turn the page and begin. Record the start time for Part 1 on the *Supervising Teacher Report*.** |

| | |
|---|---|
| | **During the test session** |
| | 1. Supervise the completion of the *Attendance Form* by ensuring that students print their name beside their test booklet number. (See sample *Attendance Form*, Appendix 3, p. 19).<br>**Note:** This is the only record connecting students' identities to their *Senior 4 Pre-Calculus Mathematics (40S) Standards Test* booklet. |
| | 2. To prepare for the distribution of Part 2 of the test, match-up each Part 2 test booklet with the appropriate numbered *Answer/Scoring Sheet*. You may wish to insert the *Answer/Scoring Sheets* inside the front cover of the appropriate test booklet. |
| | 3. Half-way through Part 1 of the test session (end of 30 minutes), advise students of the amount of time remaining for Part 1 by making an announcement and by writing the remaining time on a board. |
| | 4. Verify that all the information on each *Demographic Data Form* is accurately printed and shaded. Place all used forms in numerical order followed by any unused forms in the labelled envelope provided. **Do not use elastic bands, staples or paper clips on these forms as they may damage the sheets.** Do not seal the envelope. |
| | 5. After 60 minutes, ask the students to place their calculators on the floor for the remainder of the test session. Students retain their Part 1 booklet for the entire testing session. Advise students they may continue working on Part 1 but they may not use their calculators. |
| | 6. Distribute Part 2 of the test and *Answer/Scoring Sheet*, but **before** doing so please note:<br>**The booklet numbers for Part 1 and 2 and the number on the *Answer/Scoring Sheet* need to be identical for each student. Advise students that as they receive Part 2 and an *Answer/Scoring Sheet* to verify that these items have the same number as their Part 1 booklet number. Tell students that their answers to the selected-response questions are to be shaded on the back of the *Answer/Scoring Sheet*.** |
| | 7. **Instruct students to turn the page and begin. Record the start time for Part 2 on the *Supervising Teacher Report*.** |
| | 8. Half-way through Part 2 of the test session, advise students of the amount of time remaining by making an announcement and by writing the remaining time on a board. |
| | **Ending the test session** |
| | 1. **Record the time students finish writing the test on the *Supervising Teacher Report*.** Do not include extra time given for adaptations. |
| | 2. Have students place their *Answer/Scoring Sheets* inside the front cover of the Part 1 test booklet. Collect the test booklets from each student. |
| | 3. As students submit their test materials, ensure that each test booklet number matches the information on the *Answer/Scoring Sheet*, the *Attendance Form* and the *Demographic Data Form* (e.g., cross-reference student name and test booklet number). Each student must have the same booklet number on all test materials. |
| | 4. Place a check "✔" under Part 1 and Part 2 of the *Attendance Form* for booklets submitted, or indicate the reason why a booklet is not submitted for marking (see Appendix 3, p. 19). |

| | |
|---|---|
| | 5. Stack the test booklets in numerical order. |
| | 6. If marking is conducted in the school, leave the *Answer/Scoring Sheets* inside the front cover of each Part 1 booklet. After marking, place all used *Answer/Scoring Sheets* in numerical order followed by any unused forms. Place these forms in the labelled envelope provided. Do not seal the envelope. |
| | 7. If marking is to be conducted at a central location, place all used *Answer/Scoring Sheets* in numerical order followed by any unused forms. Place these forms in the labelled envelope provided. Do not seal the envelope. |
| | 8. Complete the *Supervising Teacher Report*. Note any unusual circumstances on this report. |
| | 9. Ensure that the number of check marks in the "Booklet Submitted to LMC for Marking" column on the *Attendance Forms* matches your count of the booklets and *Answer/Scoring Sheets* for submission. |
| | 10. Complete and check the *Student Registration Update* form. Use the reason codes (Appendix 4, p. 20) provided. |
| | 11. Make one or two copies of the *Attendance Form(s)* and *Supervising Teacher Report(s)*. One copy must be retained by the school and the other forwarded to the local marking site, if a centralized marking model is used. |
| | 12. Collect the *Principal Feedback Form*. |
| | 13. Complete the *Supervising Teacher Feedback Form*. |
| | 14. Return all test booklets and other test-related materials to the secure area in the school until commencement of marking or until materials are to be forwarded to the local marking site. |
| | 15. Send materials according to the instructions on the following pages. |

## Submitting Materials on June 10, 2003 (the end of the test)

It is essential that the following materials are packaged and forwarded to the local marking site immediately after completion of the test if a centralized model is used.

- **All used** test booklets;
- **All used** *Answer/Scoring Sheets*;
- Copy of the *Attendance Forms*; and
- Copy of the *Supervising Teacher Report*.

All other documentation should be verified and returned to the Assessment and Evaluation Branch after the test session.

- *Attendance Forms*;
- **All used** *Demographic Data Forms* (in envelope provided);
- *Student Registration Update Form*;
- *Supervising Teacher Report*;
- *Supervising Teacher Feedback Form*;
- *Principal Feedback Form*; and
- **All** damaged or misprinted booklets.

**Schools in the City of Winnipeg** return their envelopes to the Assessment and Evaluation Branch by calling Mid-Canada Courier (989-5630) and arranging pick-up between **1:00 p.m. and 4:00 p.m.** on **June 10, 2003**. The person placing the call should quote account number E901, provide the school name, address and the name of the contact person.

**Schools outside the City of Winnipeg** return their envelopes to the Assessment and Evaluation Branch by using the Xpresspost Flexipack envelope provided and taking it to the post office.

## Submitting Materials by June 20, 2003 (after local marking)

When local marking is done at a central location, the Local Marking Co-ordinator is responsible for sending the materials to Assessment and Evaluation. When local marking is done at the classroom level, the teacher is responsible for sending the materials to Assessment and Evaluation. NEW!

After local marking has been completed, prepare a package for return to Assessment and Evaluation no later than June 20, 2003.

The package must include:

- **All** *Answer/Scoring Sheets*;
- Test booklets requested by the department for the feedback marking;
- *Sample Request List*; and
- *Local Marking Co-ordinator Feedback Form*.

**Jurisdictions within the City of Winnipeg** send packages directly to the Assessment and Evaluation Branch via **Mid-Canada Courier (989-5630)**. The person placing the call should quote **account number E901**, give the jurisdiction name, address, and the name of the contact person.

**Jurisdictions outside the City of Winnipeg** return their package by Xpresspost. It is very important to package all materials in the **one Xpresspost Flexipack** envelope provided whenever possible.

After the package is sealed, deliver the package to the local post office.

We have provided sufficient materials for the return of **one** envelope/package. If you must send a second envelope/package, photocopy the mailing address label and affix the copy to the second package. Note that you must use a photocopy of the mailing address label as it provides Canada Post with billing information. Ask the postal clerk for an additional bar code label to attach to the second envelope/package. Ensure that the school representative retains the bar code number in case tracing the package is required.

Return all local marking scan sheets. Payments will be made according to the number of tests marked, based on the local marking scan sheets received.

# Appendices

## Appendix 1: Test-Taking Rules

# Senior 4 Pre-Calculus Mathematics (40S) Standards Test

**The supervising teacher is expected to read these rules aloud to students at the beginning of the *Senior 4 Pre-Calculus Mathematics (40S) Standards Test*:**

- Check the title of your test booklet carefully.
- During this test session, do not proceed until instructed to do so.
- If you receive a damaged or misprinted booklet, return it to receive another booklet.
- You are expected to remain in the test room for the allotted time for Part 1 and until you complete Part 2. (You may leave before that time in exceptional circumstances such as illness, when accompanied by a supervisor.)
- At the beginning of the session you should have only a Part 1 test booklet, an HB pencil, eraser, ruler and your calculator (scientific or graphing, for Part 1 only). No other materials are permitted. Do not exchange test materials. When Part 2 is distributed you will also receive an *Answer/Scoring Sheet* to record your answers to the selected-response questions.
- Your calculator must remain on the floor throughout the administration of Part 2. You may keep your Part 1 booklet for the entire session and and continue working on Part 1 if you wish, <u>but</u> you may not use your calculator.
- You will not give assistance to, nor receive assistance from, another student. If you require something during the test, raise your hand and I will come to you.
- Do not ask questions or request explanations with regard to the content of the questions on the test.
- Do **not** identify yourself or your school, teacher or town/city in your test booklet.
- When I ask you to stop writing, put your pencil down immediately.
- You may not discard any materials. The test booklet must remain intact unless otherwise instructed. Any extra paper used must be stapled into the test booklet on the appropriate page. You may **not** leave the room with any materials pertaining to the test.

# Appendix 2: Sample Demographic Data Form

## DEMOGRAPHIC DATA FORM

Manitoba Education, Training and Youth

### INSTRUCTIONS

- Use an HB pencil only.
- Do not use ink, ballpoint, or felt tip pens.
- Shade the circle completely.
- Erase cleanly any marks you wish to change.
- Do not mark other areas on this form.

### EXAMPLES

INCORRECT CORRECT

### PROGRAM INFORMATION

E English Program

I French Immersion Program

F Français Program

NAME (Last)

NAME (First)

PROVINCIAL STUDENT (MET) NO.

BIRTH DATE

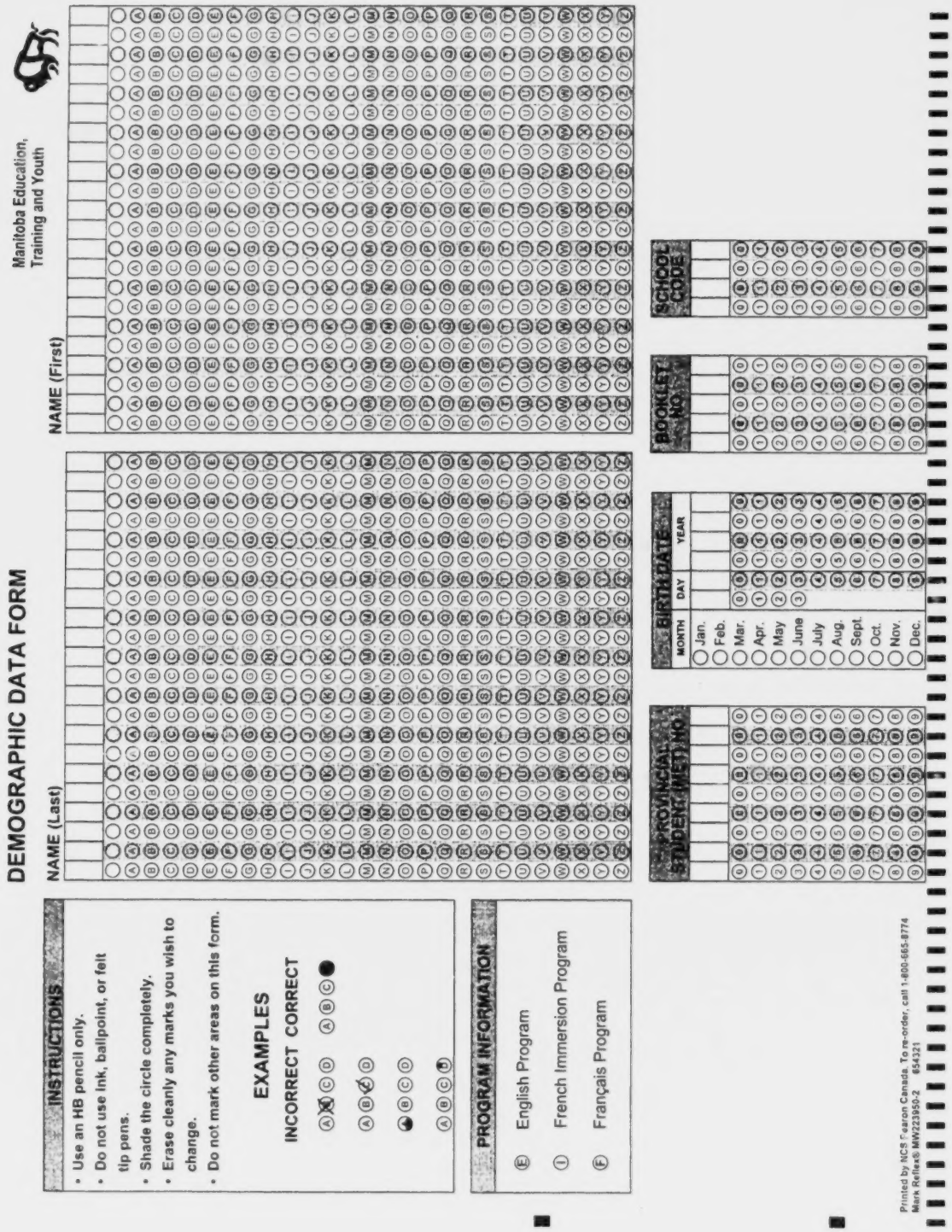
MONTH DAY YEAR

Jan
Feb
Mar
Apr
May
June
July
Aug
Sept
Oct
Nov
Dec

BOOKLET NO.

SCHOOL CODE

# Appendix 2: Sample Demographic Data Form



DEMOGRAPHIC DATA FORM

Manitoba Education, Training and Youth

NAME (Last)

NAME (First)

INSTRUCTIONS

- Use an HB pencil only.
- Do not use ink, ballpoint, or felt tip pens.
- Shade the circle completely.
- Erase cleanly any marks you wish to change.
- Do not mark other areas on this form.

EXAMPLES

INCORRECT CORRECT

PROGRAM INFORMATION

(E) English Program

(I) French Immersion Program

(F) Français Program

PROVINCIAL STUDENT (MSF) NO.

BIRTH DATE

MONTH: Jan., Feb., Mar., Apr., May, June, July, Aug., Sept., Oct., Nov., Dec.

DAY

YEAR

BOOKLET NO.

SCHOOL CODE

Printed by NCS Pearson Canada. To re-order, call 1-800-665-8774
Mark Reflex® MW223950-2 654321

# Appendix 3: Sample Attendance Form

School Number: 9999
Somewhere School
Somewhere Address
Somewhere MB R0R 0R0

Page _____ of _____

## ATTENDANCE FORM—June 2003

**SENIOR 4 PRE-CALCULUS MATHEMATICS (40S) STANDARDS TEST**

| Booklet Number | Student Name (*Please Print*) | Booklet Submitted to LMC* for Marking (*check* ✓) | |
|---|---|---|---|
| | | Part 1 | Part 2 |
| 30001 | Karen Masi | ✓ | *went home sick* |
| 30002 | Dan Carter | ✓ | ✓ |
| 30003 | Michael Smith | ✓ | ✓ |
| 30004 | DAMAGED | | |
| 30005 | Mary Jones | ✓ | ✓ |
| 30006 | Shelley Bracken (also 30008) | ✓ | ✓ |
| 30007 | UNUSED | | |
| ~~30008~~ | Used with 30006 (damaged) | | |
| ~~30009~~ | Used in another classroom | | |
| ~~30010~~ | Used in another classroom | | |

**NOTE: Record any absences on the *Student Registration Update* form.**

Myles Kaminsky

**Supervising Teacher's Name** (*please print*)

**Please ensure that copies of these forms are retained for your records. You will need this form to link your locally marked test with a student's name.**

*** Local Marking Co-ordinator**

# Appendix 4: List of Reason Codes to Record Changes in Standards Test Student Registration Numbers

(Please use these codes to complete the *Student Registration Update Form* when returning to the Assessment and Evaluation Branch in June 2003.)

| ABSENT | |
|---|---|
| 01 | arrived too late to write the test |
| 02 | did not write due to weather |
| 03 | family/personal reasons |
| 04 | medical/illness/accident |
| 05 | out-of-town activity (athletic/academic) |
| 06 | religious objections |
| 07 | student refused to write |
| 08 | unexcused |
| 09 | working |
| 19 | other (please specify) |

| WITHDRAWN | |
|---|---|
| 22 | dropped course |
| 23 | incarcerated/court |
| 24 | medical/illness/accident |
| 25 | family/personal reasons |
| 26 | poor attendance/progress |
| 27 | quit school |
| 29 | other (please specify) |

| INCOMPLETE | |
|---|---|
| 40 | continuing course/program next semester |
| 41 | no work completed |
| 49 | other (please specify) |

| ERRORS IN REGISTRATION | |
|---|---|
| 20 | changed program |
| 21 | deceased |
| 50 | already has credit |
| 51 | auditing the course |
| 52 | exchange student |
| 53 | registered twice |
| 54 | quarter system projected registration |
| 55 | registered in wrong grade |
| 59 | other (please specify) |

| EXEMPTIONS | |
|---|---|
| 60 | emotional/psychological condition or circumstances |
| 61 | "M" credit designation |
| 62 | "E" credit designation |
| 63 | "phase d'accueil—bloc intensif" designation |
| 69 | other (please specify) |

| DROP INS | |
|---|---|
| 70 | forgotten on original test registration |
| 71 | late registration to course |
| 72 | transferred in from another school/province/country |
| 73 | rewrite (not currently enrolled) |
| 79 | other (please specify) |

| MIGRANCY | |
|---|---|
| 90 | transferred to another school within the province |
| 91 | transferred to another school outside of the province |
| 99 | other (please specify) |

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# Senior 4 Pre-Calculus Mathematics (40S) Standards Test

## Part 1

June 2003

Manitoba

Building for the Future

# Pre-Calculus Mathematics (40S)
# Standards Test

## DESCRIPTION

**Total Possible Marks: 100** **Time: 3 hours**

| Part | Description | Time | Value |
|---|---|---|---|
| **Part 1** | • **Open Response:** 11 questions worth a total of 35 marks<br>Calculators (scientific or graphing) are permitted. | 60 minutes | 35% |
| **Part 2** | • **Selected Response:** 15 questions worth 1 mark each<br>• **Restricted Response:** 15 questions worth 1 mark each<br>• **Open Response:** 10 questions worth a total of 35 marks<br>Calculators are not permitted. | 120 minutes | 65% |

## GENERAL DIRECTIONS

- Fill in the demographic form accurately and completely.
- Formulas are given on the tear-out "Formula Sheet" at the beginning of this test booklet. Please do not tear out any pages other than the "Formula Sheet".
- Read all instructions carefully.
- The blank pages at the back of each booklet may be used for rough work. No marks will be given for work done on these pages.
- Note that diagrams and graphs provided in the test booklets may not be drawn to scale.
- When Part 2 is distributed, you may keep Part 1 in case you wish to continue working on Part 1 questions (without a calculator).

# Pre-Calculus Mathematics (40S) Formula Sheet

$S = r\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$

$\tan 2\alpha = \dfrac{2\tan\alpha}{1 - \tan^2\alpha}$

$P(n,r)$ or ${}_nP_r = \dfrac{n!}{(n-r)!}$

$C(n,r)$ or ${}_nC_r = \dfrac{n!}{r!(n-r)!}$

$t_{k+1} = {}_nC_k a^{n-k} b^k$

$A = P\left(1 + \dfrac{r}{n}\right)^{nt}$

$A = Pe^{rt}$

$e \approx 2.71828$

$\log_a(MN) = \log_a M + \log_a N$

$\log_a\left(\dfrac{M}{N}\right) = \log_a M - \log_a N$

$\log_a(M^n) = n\log_a M$

$\log_a M = \dfrac{\log_b M}{\log_b a}$

$(x-h)^2 + (y-k)^2 = r^2$

$\dfrac{(x-h)^2}{a^2} + \dfrac{(y-k)^2}{b^2} = 1, \quad a > b$

$\dfrac{(x-h)^2}{b^2} + \dfrac{(y-k)^2}{a^2} = 1, \quad a > b$

$\dfrac{(x-h)^2}{a^2} - \dfrac{(y-k)^2}{b^2} = 1$

$\dfrac{(y-k)^2}{a^2} - \dfrac{(x-h)^2}{b^2} = 1$

$(y-k) = a(x-h)^2$

$(x-h) = a(y-k)^2$

$t_n = t_1 r^{n-1}$

$S_n = \dfrac{t_1(1-r^n)}{1-r} = \dfrac{t_1(r^n-1)}{r-1}$

$S_n = \dfrac{t_1 - t_n r}{1-r}$

$S_\infty = \dfrac{t_1}{1-r}, \quad |r| < 1$

# Open-Response Questions

## Instructions

- There are 11 questions for a total of 35 marks.
- Calculators (scientific or graphing) are allowed for this part of the test.
- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Graphing calculator solutions should include an explanation of how your final answer is obtained.
- Your solutions should be neat, organized and clear.
- If any curve contains asymptotes, the asymptotes should be included in the graph.

DO NOT USE

# Open-Response Questions

1. Solve for $x$. Express your answer correct to 3 decimal places. 101

$$e^x = 8^{1-x}$$

(4 marks)

2. Simplify the following expression as much as possible: 102

$$(\sin x + \cos x)^2 - \sin 2x$$

(3 marks)

DO NOT USE

3. Find the numerical value of: 103

$$\frac{\log_3 9}{\log_9 3}$$

(2 marks)

4. Solve the equation for $x$. State all solutions in radian measure, correct to 3 decimal places. 104

$$\tan x = 3$$

(3 marks)

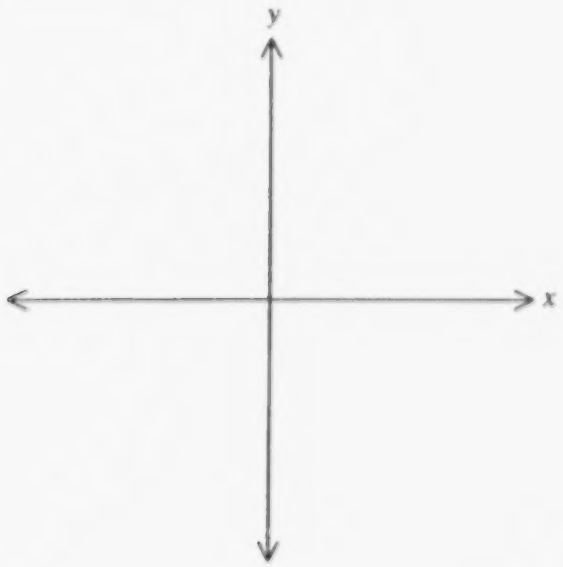
DO NOT USE

5. a) Write the equation $x^2 - 4y^2 - 16y = 0$ in standard form. 105

(2 marks)

b) Sketch a clearly labelled graph of the above equation. 106

(2 marks)



DO NOT USE

6. Consider the following digits: 0, 1, 2, 3, 4, 5. How many three-digit positive integers can be formed if the three digits are **different** and the number is **even**? 107

(3 marks)

7. There are 9 different gifts placed on a table. 108
They are to be divided amongst Anna, Betty and Connie so that each receives 3 gifts.
In how many ways can this be done?

(3 marks)

DO NOT USE

8. A committee is formed by randomly selecting 2 people from a group of 6 men and 4 women. What is the probability that both are women? 109

(3 marks)

9. Consider the expansion of $\left(x^2 + \frac{2}{x}\right)^{10}$.

a) How many terms are in this expansion? 110

(1 mark)

b) Simplify the term which contains $x^{11}$. 111

(3 marks)

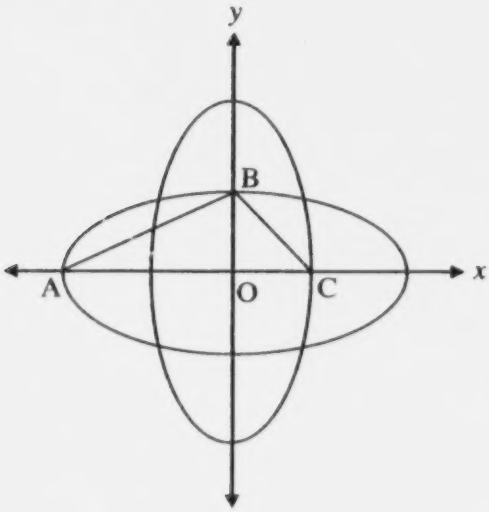
DO NOT USE

112

10. The sketches of the ellipses with equations $\frac{x^2}{9}+\frac{y^2}{16}=1$ and $\frac{x^2}{25}+\frac{y^2}{4}=1$ are given below. (The ellipses are not necessarily drawn to scale.)

Find the area of $\Delta ABC$.

(3 marks)



DO NOT USE

113

11. A circle has the equation $x^2 + y^2 + 6x - 10y + k = 0$.
Find the value of $k$ if the radius is 5.

(3 marks)

**NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE**

**NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE**

**NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE**

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# Senior 4 Pre-Calculus Mathematics (40S) Standards Test

## Part 2

June 2003

Manitoba

Building for the Future

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# Selected-Response Questions

## Instructions

- There are 15 selected-response questions.
- Each question is worth 1 mark.
- Record your answers on the sheet provided.
- Read each question carefully.
- Examine all the choices before you decide on your answer.
- You may use the spaces beside each question for rough work.
- Answer all questions.
- Provide only one answer per question.
- There is no penalty for guessing.
- Calculators (scientific or graphing) are **not** allowed for this part of the test.

## Selected-Response Questions

12. If $\sin\theta < 0$ and $\cos\theta > 0$ then $\theta$ terminates in quadrant:

    a) I

    b) II

    c) III

    d) IV

13. A reflection of the graph of any function $y = f(x)$ in the $y$-axis is:

    a) $y = |f(x)|$

    b) $y = f^{-1}(x)$

    c) $y = -f(x)$

    d) $y = f(-x)$

14. If $A = \sin 70°$, $B = \sin 170°$ and $C = \sin 275°$ then:

    a) $A > B > C$

    b) $C > B > A$

    c) $A > C > B$

    d) $C > A > B$

15. The number of ways of arranging 10 people in a circle is:

a) 9!

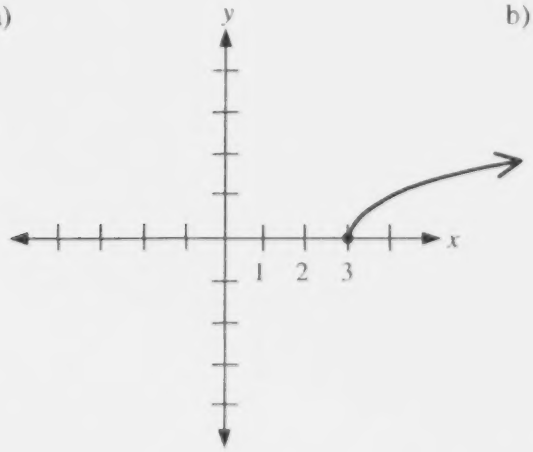
b) 10!

c) 11!

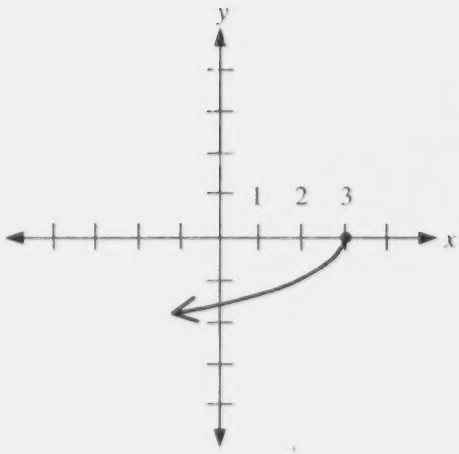
d) $\frac{9!}{2}$

16. A sketch of the graph of $f(x) = -\sqrt{x+3}$ is:

a)




b)

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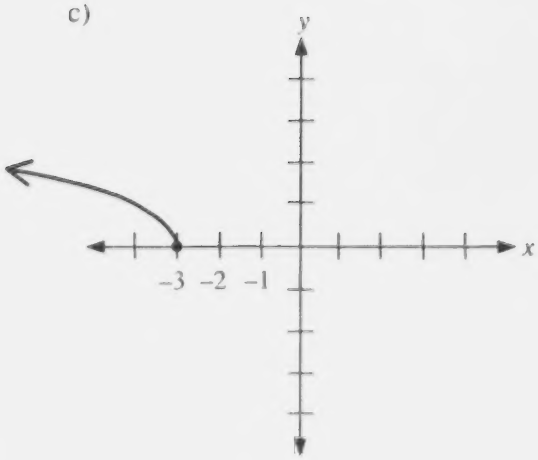


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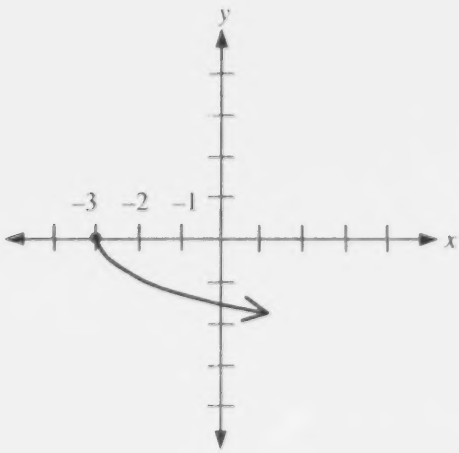
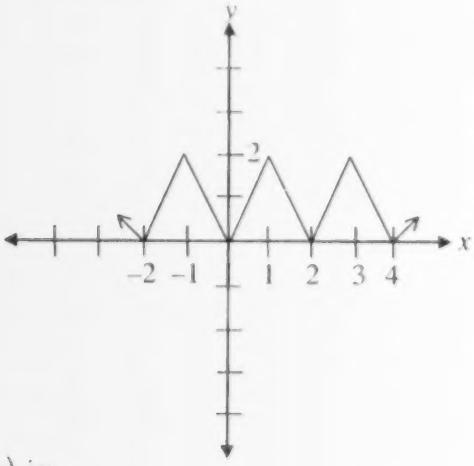
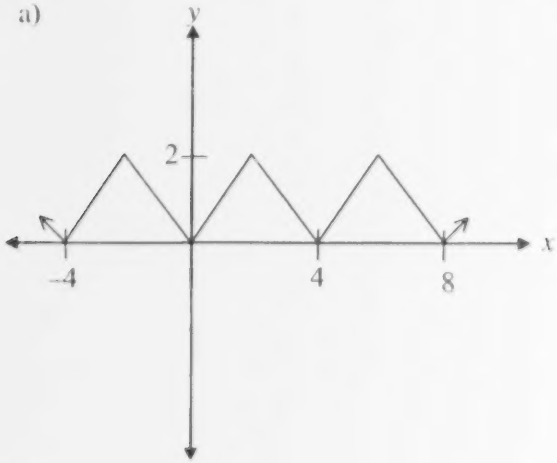


17. A sketch of $y = f(x)$ is given below.

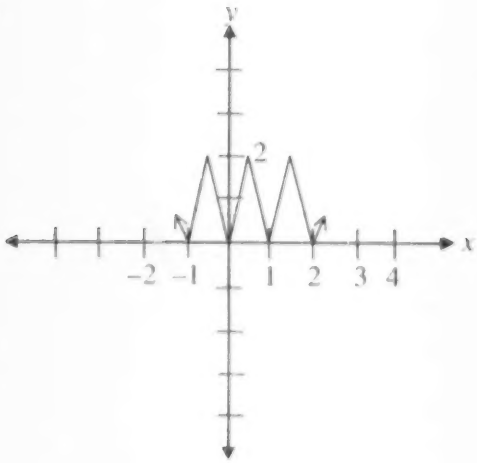



The graph of $y = 2f(x)$ is:

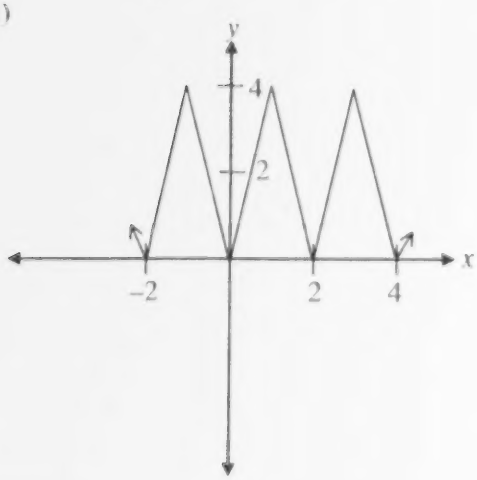
a)



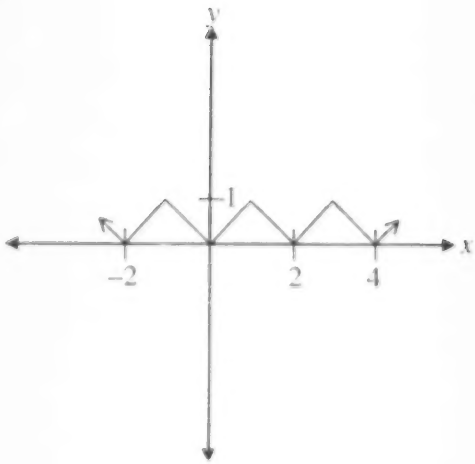

b)




c)

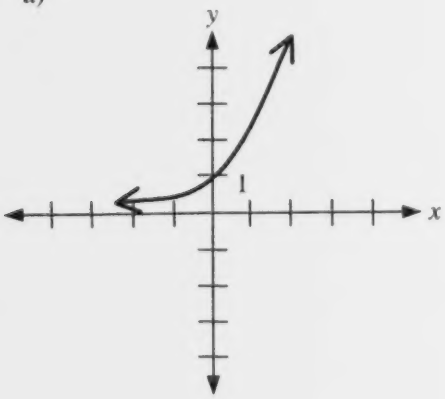



d)

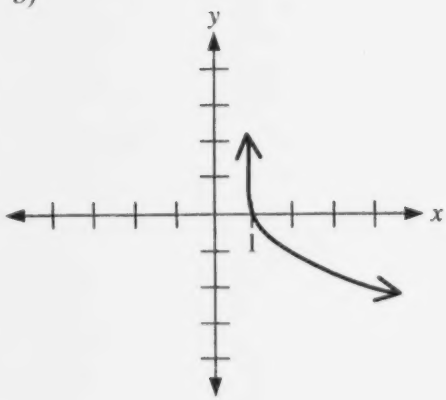


18. The graph of $g(x) = \dfrac{1}{\ln x}$ is:

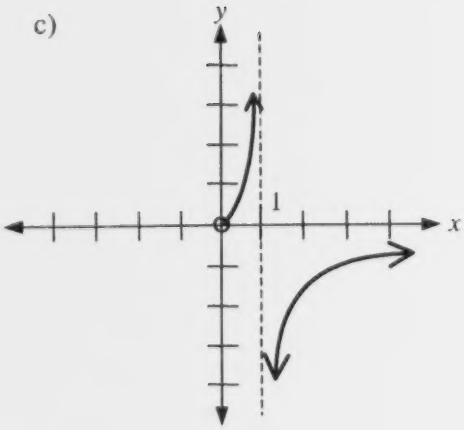
a)



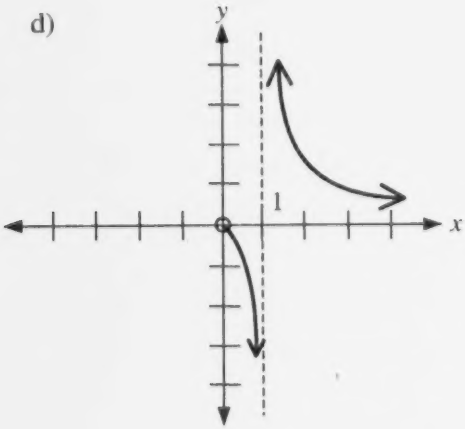

b)




c)




d)




19. The expression $\sin 4x \cos 3x - \cos 4x \sin 3x$ is equivalent to:

a) $\cos x$

b) $\sin x$

c) $\cos 7x$

d) $\sin 7x$

20. The expression $\dfrac{\log x^6}{\log x^3}$ is equivalent to:

a) 2

b) 3

c) $x^2$

d) $x^3$

21. The expression $\log 1 + \log 2 + \log 3 + \log 4 + \log 5$ is equivalent to:

a) $\log 15$

b) $\log(5!)$

c) 120

d) $10^{15}$

22. The expression $\log A - \dfrac{1}{2}\log B + 3\log C$ is equivalent to:

a) $\log\left(\dfrac{A}{\sqrt{B}\left(C^3\right)}\right)$

b) $\log\left(\dfrac{AC^3}{\sqrt{B}}\right)$

c) $\log\left(\dfrac{A(3C)}{2B}\right)$

d) $\log\left(A - \dfrac{1}{2}B + 3C\right)$

23. The domain of $f(x) = \ln(\ln x)$ is:

a) $(0, \infty)$

b) $[0, \infty)$

c) $(1, \infty)$

d) $[1, \infty)$

24. The sum to infinity of the geometric series $-27 + 9 - 3 + \ldots$ is:

a) $-\frac{81}{4}$

b) $-21$

c) $-\frac{81}{2}$

d) undefined because the series diverges

25. A geometric mean between 8 and 18 is:

a) $\frac{18}{8}$

b) $\sqrt{26}$

c) 12

d) 13

26. Which one of the following is a geometric sequence?

a) $2, 4, 6, \ldots$

b) $\frac{1}{4}, -\frac{1}{2}, 1, \ldots$

c) $\sin 0°, \sin 90°, \sin 180°, \ldots$

d) $\log x, \log x^2, \log x^3, \ldots$

# Restricted-Response Questions

## Instructions

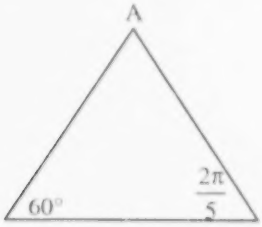
- There are 15 restricted-response questions.
- Each question is worth 1 mark.
- Calculators are NOT allowed for this part of the test.
- Write your final answer in the space provided.
- Work need not be shown in order to obtain full marks for a correct answer. However, in the case of an incorrect final answer, part marks may be awarded only if work is shown.

DO NOT USE

## Restricted-Response Questions

27. Find the measure of $\angle A$. Your answer may be stated in radians or degrees. 114

(1 mark)




28. There are 8 points placed on a circle. Triangles are formed by using 3 of these points as vertices. How many different triangles are determined by these 8 points? (Leave your answer in factorial form.) 115

(1 mark)

29. Find the numerical value of:

$$\sec \frac{\pi}{3}$$ 116

(1 mark)

DO NOT USE

30. If $f(x) = \log_{16} x$, find the value of $f(4)$. 117

(1 mark)

31. If $x = e^{2\ln 3}$ then find the value of $x$ as a whole number. 118

(1 mark)

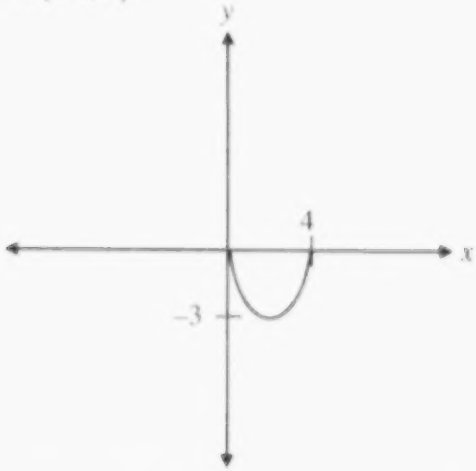
32. The binomial $(x+y)^9$ is expanded using the Binomial Theorem. What is the value of $a$ in the term ${}_9C_6 x^a y^6$? 119

(1 mark)

DO NOT USE

33. The sketch of $f(x)$, in the interval [0, 4] is given below. If $f(x)$ is an odd function, sketch $f(x)$ in the interval [– 4, 0]. 120

(1 mark)

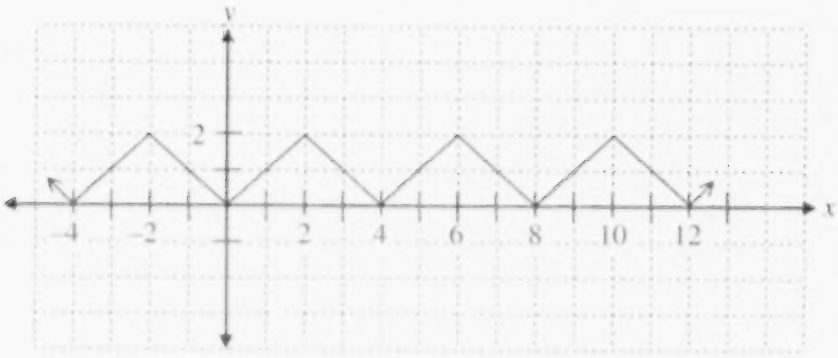



34. Find the number of permutations of all the letters of the word PEPPER. 121
State your answer as a whole number.

(1 mark)

35. The graph of the periodic function $f(x)$ is given below. 122

(1 mark)




The value of $f(98)$ is: _____

DO NOT USE

36. The probabilities of the independent events A and B are P(A) = 0.5 and P(B) = 0.6. Find the value of P(A or B). 123

(1 mark)

37. The probability that Barry passes History is $\frac{3}{4}$. The probability that he passes Science is $\frac{4}{5}$. Find the probability that Barry passes History but fails Science. 124

(1 mark)

38. $P(\theta)$ is the terminal point on the unit circle. The coordinates of $P(20°)$ are $(a, b)$. What are the coordinates of $P(340°)$ in terms of $a$ and $b$? 125

(1 mark)

DO NOT USE

39. Find a numerical expression for the $50^{th}$ term of the geometric sequence: 126

$2^4, 2^6, 2^8, \ldots$

(1 mark)

40. The $x$-intercept of $y = f(x)$ is 5. 127

Find the $x$-intercept of $y = 2f(x-3)$.

(1 mark)

41. Find the range of the function: 128

$f(x) = \cos x + \cos(-x)$

(1 mark)

# Open-Response Questions

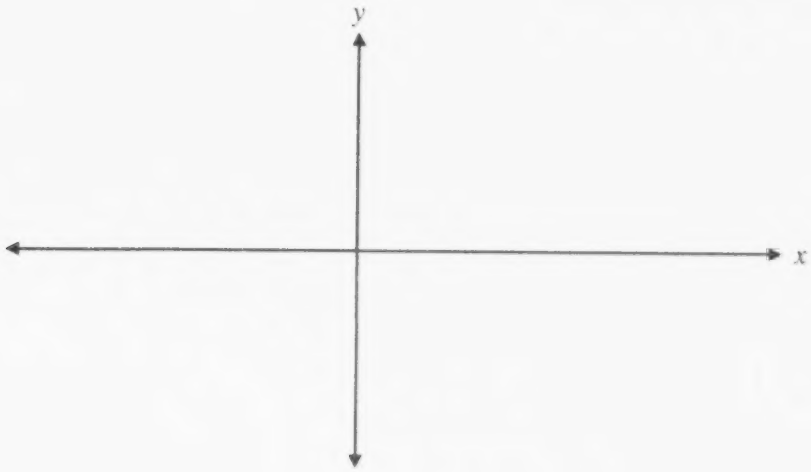
## Instructions

- There are 10 questions worth a total of 35 marks.
- Calculators are NOT allowed for this part of the test.
- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Your solutions should be neat, clear and well organized.
- If any curve contains asymptotes, the asymptotes should be included in the graph.

DO NOT USE

42. a) Sketch a clearly labelled graph of at least one period of the curve $y = \cos x$. 129

(1 mark)




b) State the $y$-intercept of $y = \cos x$.

(1 mark)

$y$-intercept = _____ 130

DO NOT USE

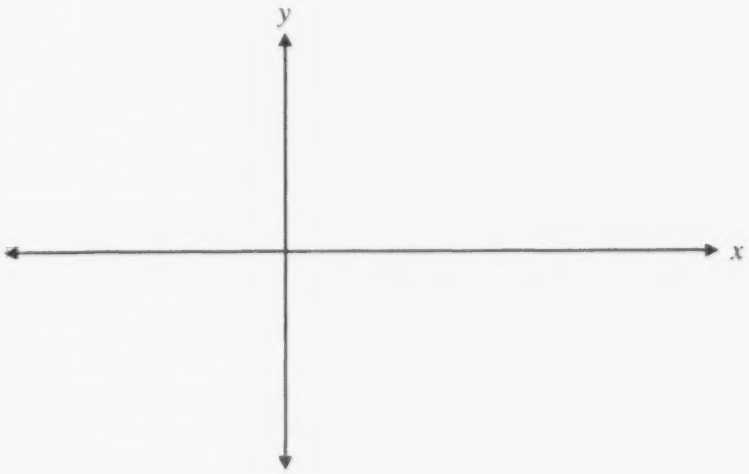
43. a) Find the $x$-intercept(s) of $f(x) = |x^3 - 8|$. 131

(1 mark)

$x$-intercept(s) = _____

b) Sketch a clearly labelled graph of $f(x) = |x^3 - 8|$. 132

(2 marks)



DO NOT USE

44. Solve for $x$: 133

$$\log_2(2-x) = 1 - \log_2(3-x)$$

(4 marks)

45. Solve for $x$ in the interval $[0, 2\pi]$: 134

$$\cos^2 x = \sin^2 x + \sin x$$

(4 marks)

DO NOT USE

46. a) Prove the identity $\tan x = \dfrac{\sin 2x}{1+\cos 2x}$. 135

(3 marks)

b) Use the identity $\tan x = \dfrac{\sin 2x}{1+\cos 2x}$ to find an exact value of $\tan\left(\dfrac{\pi}{8}\right)$, in simplified form. 136

(2 marks)

DO NOT USE

47. The angle $\alpha$ is in quadrant II and $\sin\alpha = \frac{3}{5}$. 137

a) Find the exact numerical value of $\cos\alpha$.

(1 mark)

b) Find the exact numerical value of $\cos(\alpha - 60°)$. 138

(3 marks)

48. A bag contains 4 blue marbles and 6 red marbles. 139
A marble is selected at random from this bag and is replaced with a marble of the other colour.
A second marble is selected at random from the new mixture of 10 marbles in this bag.
Find the probability that the second marble selected is red.

(4 marks)

DO NOT USE

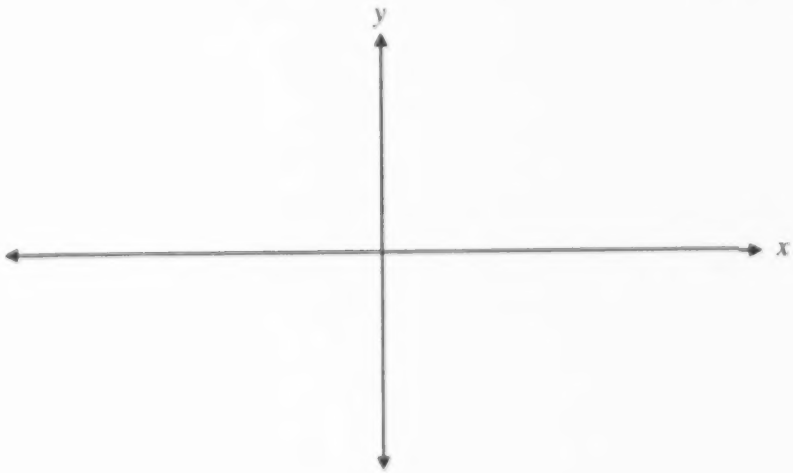
49. Solve for $n$: 140

$$\frac{(n+4)!}{(n+2)!} = 2$$

(2 marks)

50. a) Sketch a clearly labelled graph of at least one period of $f(x) = \sin(\pi x) + 2$. 141

(2 marks)




b) The period of $f(x) = \sin(\pi x) + 2$ is _____ 142

(1 mark)

c) The range of $f(x) = \sin(\pi x) + 2$ is _____ 143

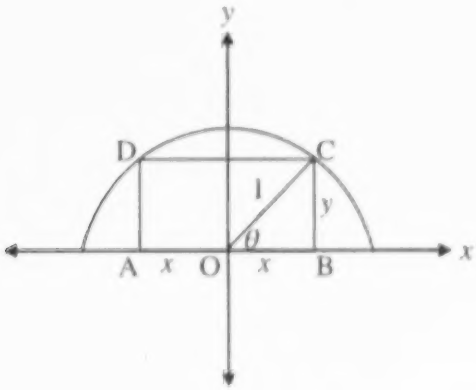
(1 mark)

DO NOT USE

144

51. C and D are points on the unit circle and A and B are on the $x$-axis. If $\angle COB = \theta$, prove that the area of rectangle ABCD $= \sin 2\theta$.

(3 marks)



**NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE**

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# Senior 4 Pre-Calculus Mathematics (40S) Standards Test

# Marking Guide

June 2003

Manitoba

Building for the Future

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Manitoba Education and Youth,
Assessment and Evaluation Branch,
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Part 2: Open-Response Questions ..... 51

# General Marking Instructions

**Please make no marks in the student test booklets.** (If the booklets have marks in them, the marks need to be removed by department staff prior to feedback marking should the booklet be selected.)

Please ensure that:

- the student test booklet number and the number on the *Answer/Scoring Sheet* are identical;
- **only a pencil is used to complete the *Answer/Scoring Sheets*, by both students and markers**;
- the totals of each of the four parts are written at the bottom;
- each student's final result is recorded, by test booklet number, on the corresponding ***Answer/Scoring Sheet***;
- the grand total is written in the appropriate location;
- the *Answer/Scoring Sheet* is complete; and
- a photocopy has been made for school records.

Once marking is completed, please forward the ***Answer/Scoring Sheets*** to Manitoba Education and Youth in the envelope provided (for more information see the *Administration Manual for Supervising Teachers*).

## Scoring the Selected-Response Questions

Please follow the instructions on the ***Answer Key for Selected-Response Questions*** on page 5.

## Scoring the Restricted-Response and Open-Response Questions

The remainder of Senior 4 Pre-Calculus Mathematics (40S) Standards Test is composed of Restricted-Response questions and Open-Response questions. Restricted-Response questions are worth 1 mark each and Open-Response questions are worth 2 to 5 marks each.

Each question is designed to elicit a well-defined response according to the associated specific learning outcome(s) and relevant mathematical processes. Their purpose is to determine whether a student meets the standards for the course as they relate to the knowledge and skills associated with the question.

To receive full marks, a student's response must be complete and correct. Otherwise, 1 mark must be deducted for each concept error, and ½ mark must be deducted for a calculation or arithmetic error. Where alternative answering methods are possible, the *Marking Key* attempts to address the most common solutions.

## Assistance

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education and Youth at the earliest opportunity to advise and/or seek assistance. If required, we will connect you to an advisor. Also, please be sure to notify us before making any modifications to the answer keys or scoring rubrics. Contact person: Neil Vadeboncoeur at 945-6715 or Chantal Bérard at 945-7590 or toll-free at 1 800 282-8069, extension 6715 or 7590.

# SCORING GUIDELINES

## *Answer Key for Selected-Response Questions*

# Answer Key for Selected-Response Questions

If you are using this sheet by punching out holes corresponding to the correct answers and overlaying this sheet on the *Answer/Scoring Sheet*, then

1) check the student's responses for multiple bubbles first (questions with multiple bubbles are to be scored as "0");

2) overlay this page on the *Answer/Scoring Sheet*; and

3) count the number of correct responses, excluding questions with multiple responses, if any.

You may also score the Selected-Response questions by making a transparency of this page and overlaying it on the *Answer/Scoring Sheet*.

Remember to write the total score for the Selected-Response questions at the bottom of the *Answer/Scoring Sheet*.

**SELECTED RESPONSE QUESTIONS / QUESTIONS À RÉPONSE CHOISIE**

**Fill in the best answer for each question. / Choisir la meilleure réponse pour chaque question.**

| | | | | | |
|---|---|---|---|---|---|
| 12 | Ⓐ Ⓑ Ⓒ **Ⓓ** | 17 | Ⓐ Ⓑ **Ⓒ** Ⓓ | 22 | Ⓐ **Ⓑ** Ⓒ Ⓓ |
| 13 | Ⓐ Ⓑ Ⓒ **Ⓓ** | 18 | Ⓐ Ⓑ Ⓒ **Ⓓ** | 23 | Ⓐ Ⓑ **Ⓒ** Ⓓ |
| 14 | **Ⓐ** Ⓑ Ⓒ Ⓓ | 19 | Ⓐ **Ⓑ** Ⓒ Ⓓ | 24 | **Ⓐ** Ⓑ Ⓒ Ⓓ |
| 15 | **Ⓐ** Ⓑ Ⓒ Ⓓ | 20 | **Ⓐ** Ⓑ Ⓒ Ⓓ | 25 | Ⓐ Ⓑ **Ⓒ** Ⓓ |
| 16 | Ⓐ Ⓑ Ⓒ **Ⓓ** | 21 | Ⓐ **Ⓑ** Ⓒ Ⓓ | 26 | Ⓐ **Ⓑ** Ⓒ Ⓓ |

| Selected-Response | | | |
|---|---|---|---|
| *Question* | *Outcome* | *Question* | *Outcome* |
| 12 | A2 | 20 | D5 |
| 13 | B3 | 21 | D5 |
| 14 | A6 | 22 | D5 |
| 15 | E2 | 23 | D7 |
| 16 | B1, B3 | 24 | H3 |
| 17 | B2 | 25 | H1 |
| 18 | B4, D7 | 26 | H1 |
| 19 | C2 | | |

# SCORING GUIDELINES

## *Marking Key*
## including student exemplars

# Part 1—Open-Response Questions

## Question No. 1

**Outcome: D7**

### Question

1. Solve for $x$. Express your answer correct to 3 decimal places.

$$e^x = 8^{1-x}$$

4 marks

### Solution

| | |
|---|---|
| $x \ln e = (1-x)\ln 8$ | 1 mark |
| $x = \ln 8 - x \ln 8$ | ½ mark |
| $x + x\ln 8 = \ln 8$ | |
| $x(1+\ln 8) = \ln 8$ | 1 mark |
| $x = \dfrac{\ln 8}{1+\ln 8}$ | ½ mark |
| $x = 0.675$ | 1 mark |

OR

| | |
|---|---|
| $x \log e = (1-x)\log 8$ | 1 mark |
| $x \log e = \log 8 - x \log 8$ | ½ mark |
| $x \log e + x \log 8 = \log 8$ | ½ mark |
| $x(\log e + \log 8) = \log 8$ | ½ mark |
| $x = \dfrac{\log 8}{\log e + \log 8}$ | ½ mark |
| $x = 0.675$ | 1 mark |

### Note(s)

- $\log 8 = 0.90309$ $\quad \log e = 0.43429$ $\quad \ln 8 = 2.07944$
- **deduct** ½ mark if parentheses are assumed but not written in Step 1
- **deduct** ½ mark for incorrect precision (incorrect number of decimals or an error in rounding)

# EXEMPLARS

1. Solve for $x$. Express your answer correct to 3 decimal places.

$$e^x = 8^{1-x}$$

$$\cancel{x}(\ln e)^1 = 1 - x(\ln 8)$$
$$x = \ln 8 - \ln 8x$$
$$\ln 8x + x = \ln 8$$
$$x(\ln 8) = \ln 8$$
$$\underline{\underline{x = 1}}$$

| | |
|---|---|
| **Mark:** | **1 out of 4** |
| **Note(s):** | • 1½ marks for first 3 lines<br>• deduct ½ mark for missing brackets in line 1 |

# Question No. 2

**Outcome: C2**

## Question

2. Simplify the following expression as much as possible:

$$(\sin x + \cos x)^2 - \sin 2x$$

3 marks

## Solution

### Method 1

| | |
|---|---|
| $\sin^2 x + 2\sin x\cos x + \cos^2 x - \sin 2x$ | 1 mark for correct expansion |
| $= (\sin^2 x + \cos^2 x) + (\sin 2x - \sin 2x)$ | 1 mark for identity |
| $= 1 + 0$ | |
| $= 1$ | 1 mark |

### Method 2

| | |
|---|---|
| $\sin^2 x + 2\sin x\cos x + \cos^2 x - \sin 2x$ | 1 mark for correct expansion |
| $= \sin^2 x + \cos^2 x + 2\sin x\cos x - 2\sin x\cos x$ | 1 mark for identity |
| $= 1 + 0$ | |
| $= 1$ | 1 mark |

## Note(s)

- **deduct** ½ mark if $\sin^2 x$ is written as $\sin x^2$
- 1 mark for $(1 - \sin 2x)$ or $(1 - 2\sin x\cos x)$ as a final answer

# EXEMPLARS

2. Simplify the following expression as much as possible:

$$(\sin x + \cos x)^2 - \sin 2x$$

$\sin^2 x + \cos^2 x + \sin x \cos x - 2\sin x \cos x$

$\sin^2 x + \cos^2 x - \sin x \cos x$

$\sin^2 x + 1 - \sin^2 x - \sin x \cos x$

$= \underline{1 - \sin x \cos x}$

**Mark:** **2 out of 3**

**Note(s):** • deduct 1 mark for incorrect first step

# Question No. 3

**Outcome: D5**

## Question

3. Find the numerical value of:

2 marks

$$\frac{\log_3 9}{\log_9 3}$$

## Solution

### Method 1

$$\frac{\log_3 9}{\log_9 3} = \frac{2}{0.5}$$ 1½ marks $\left\{ \begin{array}{l} (\text{½ mark for } \log_3 9 = 2) \\ (\text{1 mark for } \log_9 3 = \frac{1}{2}) \end{array} \right.$

$= 4$ ½ mark

### Method 2

$$\frac{\frac{\log 9}{\log 3}}{\frac{\log 3}{\log 9}}$$ ½ mark

$$= \frac{\frac{0.95424}{0.47712}}{\frac{0.47712}{0.95424}}$$ ½ mark

$$= \frac{2}{0.5}$$ ½ mark

$= 4$ ½ mark

## Note(s)

- $\ln 9 = 2.19722$
- $\ln 3 = 1.09861$

# EXEMPLARS

3. Find the numerical value of:

$$\frac{\log_3 9}{\log_9 3}$$

$$= \frac{\frac{\log_{10} 9}{\log_{10} 3}}{\frac{\log_{10} 3}{\log_{10} 9}}$$

$$= \frac{\frac{0{,}954}{0{,}477}}{\frac{0{,}477}{0{,}954}}$$

$$= \frac{2}{2}$$

$$= 1$$

**Mark:** **1 out of 2**

**Note(s):** • concept error $\frac{0.954}{0.477} \neq \frac{0.477}{0.954}$

# Question No. 4

**Outcome: A5**

## Question

4. Solve the equation for $x$. State all solutions in radian measure, correct to 3 decimal places.

$$\tan x = 3$$

3 marks

## Solution

**Method 1**

Reference arc $= \tan^{-1} 3$

$= 1.249$ 1 mark

$x = 1.249 + 2\pi(k),\ k \in \text{Integer}$

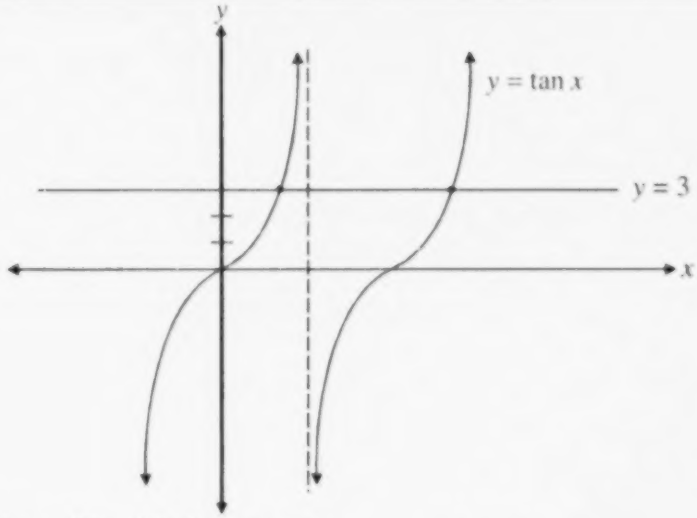
$x = 4.391 + 2\pi(k),\ k \in \text{Integer}$ 2 marks $\begin{cases} \text{1 mark for 1.249 and 4.391} \\ \text{1 mark for generalizing} \end{cases}$

OR

$x = 1.249 + k\pi,\ k \in \text{Integer}$ 2 marks

**Method 2**

Graphing Calculator Solution




Intersect $x = 1.249$ 1 mark

Intersect $x = 4.391$ 1 mark

Final Answers $\left.\begin{matrix} 1.249 + 2k\pi \\ 4.391 + 2k\pi \end{matrix}\right\} k \in I$ OR $1.249 + k\pi,\ k \in I$ 1 mark

## Note(s)

- $1.249 + \pi$ is equivalent to 4.391
- award a maximum of 2 marks for a correct solution in degrees with all degree symbols (deduct ½ mark for not including degree symbol)
  i.e., $71.565° + 180°k,\ k \in I$ or $71.565° + 360°k,\ k \in I$ and $251.565° + 360°k,\ k \in I$
- **deduct** ½ mark for not stating $k \in I$
- award a maximum of 2 marks for solutions which are not generalized

# EXEMPLARS

4. Solve the equation for $x$. State all solutions in radian measure, correct to 3 decimal places.

$$\tan x = 3$$

$\tan^{-1} 3 = 71.57$

Quad I = 71.57

Quad III = 251.57

$71.57k + 2k\pi \quad k \in \mathbb{I}$

$251.57k + 2k\pi \quad k \in \mathbb{I}$

**Mark:** ½ **out of 3**

**Note(s):**
- 1 mark for correct reference angle and deduct ½ mark for missing degree symbol

# Question No. 5

**Outcome: F2, F3**

## Question

5. a) Write the equation $x^2 - 4y^2 - 16y = 0$ in standard form. **2 marks**

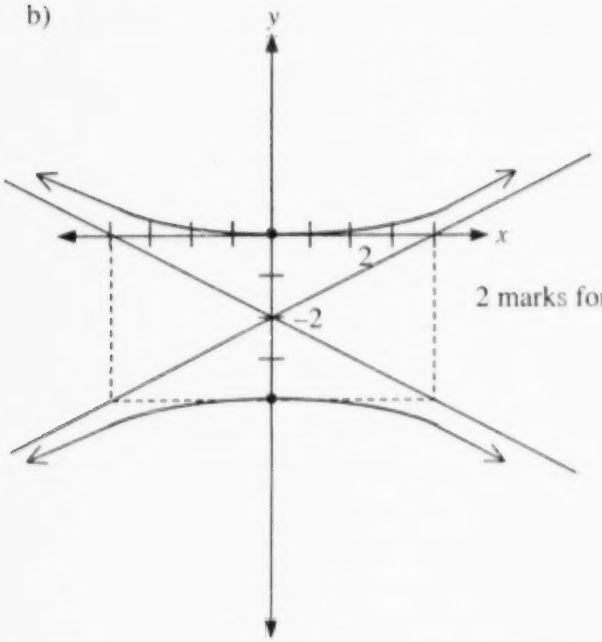
   b) Sketch a clearly labelled graph of the above equation. **2 marks**

## Solution

a)
$$x^2 - 4\left(y^2 + 4y + 4 - 4\right) = 0$$
$$x^2 - 4\left(y+2\right)^2 = -16$$
$$\frac{\left(y+2\right)^2}{4} - \frac{x^2}{16} = 1$$

2 marks for completing the square

b)




2 marks for sketch
- ½ mark for vertical hyperbola
- 1 mark for vertices
- ½ mark for asymptotes

## Note(s)

b)
- if hyperbola graph is consistent with equation in part a) give 2 marks
- if non hyperbola graph is consistent with equation in part a) give 1 mark

# EXEMPLARS

5. a) Write the equation $x^2 - 4y^2 - 16y = 0$ in standard form.

$x^2 \quad -4y^2 - 16y = 0$

$x^2 \quad -4(y^2 + 4y \quad) = 0$

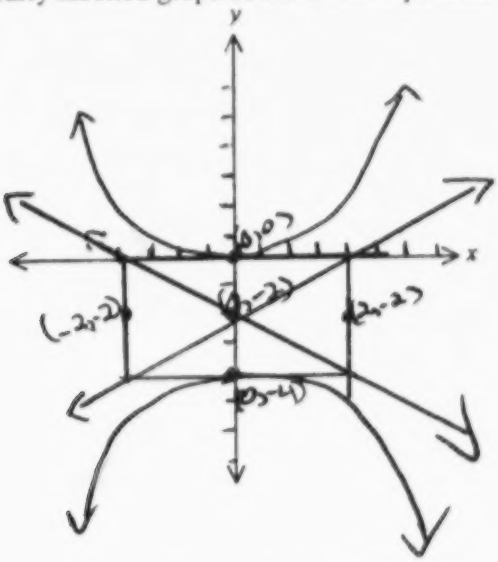
$x^2 - 4(y^2 + 4y + 4) = 16$

$\frac{x^2}{16} \quad \frac{-4(y+2)^2}{16} = \frac{16}{16}$

$\boxed{\frac{x^2}{16} \quad \frac{-(y+2)^2}{4} = 1}$ $\boxed{C\ (0,-2)}$

| | |
|---|---|
| **Mark:** | **1 out of 2** |
| **Note(s):** | • deduct 1 mark for incorrect completion of square |

b) Sketch a clearly labelled graph of the above equation.




| | |
|---|---|
| **Mark:** | **1 out of 2** |
| **Note(s):** | • deduct 1 mark for inconsistent graph |

# Question No. 6

**Outcome: E1**

## Question

6. Consider the following digits: 0, 1, 2, 3, 4, 5. How many three-digit positive integers can be formed if the three digits are **different** and the number is **even**?

3 marks

## Solution

### Method 1

Case 1: Ends with the digit 0

$$\underline{5} \cdot \underline{4} \cdot \underset{\{0\}}{\underline{1}} = 20$$

1 mark for case 1

Case 2: Ends with the digit 2 or 4

$$\underset{\{\sim 0\}}{\underline{4}} \cdot \underline{4} \cdot \underset{\{2, 4\}}{\underline{2}} = 32$$

1 mark for case 2

Number of even numbers

$= 20 + 32$
$= 52$

1 mark for consistent answer (addition of cases)

### Method 2

Case 1: All Possible 3-digit numbers

$$\underset{\{\sim 0\}}{\underline{5}} \cdot \underline{5} \cdot \underline{4} = 100$$

1 mark for case 1

Case 2: All odd 3-digit numbers

$$\underset{\{\sim 0\}}{\underline{4}} \cdot \underline{4} \cdot \underset{\{1, 3, 5\}}{\underline{3}} = 48$$

1 mark for case 2

Number of even numbers

$= 100 - 48$
$= 52$

1 mark for consistent answer (subtraction of cases)

# Question No. 6

**Outcome: E1**

## Question

6. Consider the following digits: 0, 1, 2, 3, 4, 5. How many three-digit positive integers can be formed if the three digits are **different** and the number is **even**?

3 marks

## Solution

### Method 3

Case 1: Even three-digit arrangements

$\underline{5} \cdot \underline{4} \cdot \underset{\text{even}}{\underline{3}} = 60$ — 1 mark for case 1

Case 2: Start with 0

$\underset{\{0\}}{\underline{1}} \cdot \underline{4} \cdot \underset{\substack{\text{even} \\ \text{non zero}}}{\underline{2}} = 8$ — 1 mark for case 2

$\therefore$ there are $60 - 8 = 52$ — 1 mark for consistent answer (subtraction of cases)

### Method 4

Case 1: Start with an odd digit

$\underset{\{1,3,5\}}{\underline{3}} \cdot \underline{4} \cdot \underset{\text{even}}{\underline{3}} = 36$ — 1 mark for case 1

Case 2: Start with an even digit

$\underset{\{2,4\}}{\underline{2}} \cdot \underline{4} \cdot \underset{\text{even}}{\underline{2}} = 16$ — 1 mark for case 2

There are $36 + 16 = 52$ — 1 mark for consistent answer (addition of cases)

## Note(s)

- give 1 mark for $\underset{\{-0\}}{\underline{5}} \cdot \underline{6} \cdot \underset{E}{\underline{3}} = 90$
- if cases are not identified deduct 1 mark

# EXEMPLARS

6. Consider the following digits: 0, 1, 2, 3, 4, 5. How many three-digit positive integers can be formed if the three digits are **different** and the number is **even**?

Case I 4 4 1 = 16 ways
4

Case II 4 4 1 = 16 ways
2

Case III 4 4 1 16 ways
0

16+16+16 = 48 ways

| | |
|---|---|
| **Mark:** | **2 out of 3** |
| **Note(s):** | • deduct 1 mark for incorrect case 3 (a concept error) |

# Question No. 7

**Outcome: E3**

## Question

7. There are 9 different gifts placed on a table.
They are to be divided amongst Anna, Betty and Connie so that each receives 3 gifts.
In how many ways can this be done?

3 marks

## Solution

### Method 1

| | | |
|---|---|---|
| ${}_9C_3 \cdot {}_6C_3 \cdot {}_3C_3$ | 2 marks | (1 mark for ${}_9C_3$, 1 mark for ${}_6C_3$) |
| $= 84 \cdot 20 \cdot 1 = 1680$ | 1 mark | |

### Method 2

| | |
|---|---|
| $\frac{9!}{3!3!3!}$ | 1 mark for numerator<br>1 mark for denominator |
| $= 1680$ | 1 mark |

## Note(s)

- no marks for 9! or 362880 as a complete answer
- **deduct** 1 mark if 1680 is multiplied by 3! (10080)
- award 1 mark (maximum) for ${}_9P_3 \cdot {}_6P_3 \cdot {}_3P_3$

# EXEMPLARS

7. There are 9 different gifts placed on a table.
They are to be divided amongst Anna, Betty and Connie so that each receives 3 gifts.
In how many ways can this be done?

Anna $_9C_3 = 84$

Brigitte $_6C_3 = 20$

Connie $_3C_3 = 1$

105 different ways

| | |
|---|---|
| **Mark:** | **2 out of 3** |
| **Note(s):** | • deduct 1 mark for concept error (added instead of multiplied) |

# Question No. 8

**Outcome: G5**

## Question

8. A committee is formed by randomly selecting 2 people from a group of 6 men and 4 women. What is the probability that both are women?

3 marks

## Solution

### Method 1

$$P(ww) = \frac{{}_4C_2}{{}_{10}C_2} = \frac{\frac{4!}{2!2!}}{\frac{10!}{2!8!}} = \frac{6}{45} = \frac{2}{15} \text{ or } 0.1\overline{3}$$

1 mark for numerator ${}_4C_2$
1 mark for denominator ${}_{10}C_2$
1 mark for consistent answer

### Method 2

$$P(ww) = \underset{\substack{\uparrow \\ \text{1 mark}}}{\frac{4}{10}} \cdot \underset{\substack{\uparrow \\ \text{1 mark}}}{\frac{3}{9}} = \frac{12}{90} = \frac{2}{15} \text{ or } 0.1\overline{3}$$

1 mark for consistent answer

### Method 3

$$\frac{{}_4C_2}{{}_6C_2 + {}_4C_2 + {}_6C_1 \cdot {}_4C_1}$$

1 mark for numerator
1 mark for denominator

$$\frac{\frac{4!}{2!2!}}{\frac{6!}{4!2!} + \frac{4!}{2!2!} + \frac{6!}{5!1!} \cdot \frac{4!}{3!1!}} = \frac{6}{15+6+24} = \frac{2}{15} \text{ or } 0.1\overline{3}$$

1 mark for consistent answer

## Note(s)

- $\frac{{}_4P_2}{{}_{10}P_2} = \frac{2}{15}$ award 3 marks (award 0 marks if ${}_4P_2$ and ${}_{10}P_2$ are not written as a ratio)

# EXEMPLARS

8. A committee is formed by randomly selecting 2 people from a group of 6 men and 4 women. What is the probability that both are women?

Total number: 10.

If 1st woman is removed: $\frac{4}{10} \cdot \frac{3}{9} = \frac{2}{15}$

4 women out of 10 — 3 women out of 9

If 1st woman is not removed: $\frac{4}{10} \cdot \frac{4}{10} = \frac{4}{25}$

4 women out of 10 — 4 women out of 10.

Since question does not state 1st woman is removed then we must assume 1st woman is not removed ∴ probability of selecting 2 women is $\boxed{\frac{4}{25}}$

| | |
|---|---|
| **Mark:** | **2 out of 3** |
| **Note(s):** | • incorrect interpretation |

# Question No. 9

**Outcome: E4**

## Question

9. Consider the expansion of $\left(x^2+\frac{2}{x}\right)^{10}$.

   a) How many terms are in this expansion? **1 mark**

   b) Simplify the term which contains $x^{11}$. **3 marks**

## Solution

a) 11 1 mark for answer

**Method 1**

b) Pattern of exponents is

$$\left.\begin{array}{lll} \left(x^2\right)^{10} & \left(x^2\right)^9\left(\frac{2}{x}\right) & \left(x^2\right)^8\left(\frac{2}{x}\right)^2 \\ x^{20} & x^{17} & x^{14} \\ \text{The exponents are decreasing by 3.} & & \\ \therefore\ x^{20}, x^{17}, x^{14}, x^{11}, x^8, x^5 & & \end{array}\right\} \text{1 mark for method}$$

fourth term $= {}_{10}C_3\left(x^2\right)^7\left(\frac{2}{x}\right)^3$ 1 mark

$= 960x^{11}$ 1 mark

**Method 2**

b)

$= {}_{10}C_3\left(x^2\right)^7\left(\frac{2}{x}\right)^3$ 2 marks

$= 960x^{11}$ 1 mark

# Question No. 9

**Outcome: E4**

## Question

9. Consider the expansion of $\left(x^2+\frac{2}{x}\right)^{10}$.

   a) How many terms are in this expansion? — 1 mark

   b) Simplify the term which contains $x^{11}$. — 3 marks

## Solution

### Method 3

b)

$t_{k+1} = {}_{10}C_k\left(x^2\right)^{10-k}\left(\frac{2}{x}\right)^k$

$\left(x^2\right)^{10-k}\left(\frac{1}{x}\right)^k = x^{11}$ —numerical coefficient may be dropped

$\frac{x^{20-2k}}{x^k} = x^{11}$

$x^{20-3k} = x^{11}$

$20-3k = 11$

$k = 3$ 1 mark

$t_4 = {}_{10}C_3\left(x^2\right)^7\left(\frac{2}{x}\right)^3$ 1 mark

$= 960x^{11}$ 1 mark

## Note(s)

- **deduct** ½ mark for a final answer of 960 in part b)
- no part marks in part a)

# EXEMPLARS

9. Consider the expansion of $\left(x^2 + \frac{2}{x}\right)^{10}$.

a) How many terms are in this expansion?

11

**Mark: 1 out of 1**

b) Simplify the term which contains $x^{11}$.

$${}_{10}C_3\left(x^2\right)^7\left(\frac{2}{x}\right)^3$$

$$= 120\left(x^{14}\right)\left(\frac{2}{x}\right)^3$$

$$= 240x^{11}$$

**Mark: 2 out of 3**

**Note(s):** • deduct 1 mark for concept error in b) $120(2)^3 \neq 120(2)(1)^3$

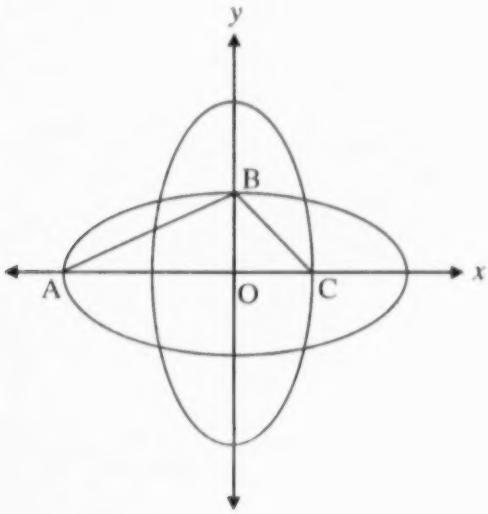
# Question No. 10

**Outcome: F3**

## Question

10. The sketches of the ellipses with equations $\frac{x^2}{9}+\frac{y^2}{16}=1$ and $\frac{x^2}{25}+\frac{y^2}{4}=1$ are given below.
(The ellipses are not necessarily drawn to scale.)

Find the area of $\Delta ABC$.




3 marks

## Solution

| | |
|---|---|
| AO = 5 | ½ mark |
| OC = 3 | ½ mark |
| Base AC = 8 | ½ mark |
| Height OB = 2 | ½ mark |
| Area = $\frac{1}{2}(8)(2)=8$ | 1 mark |

## Note(s)

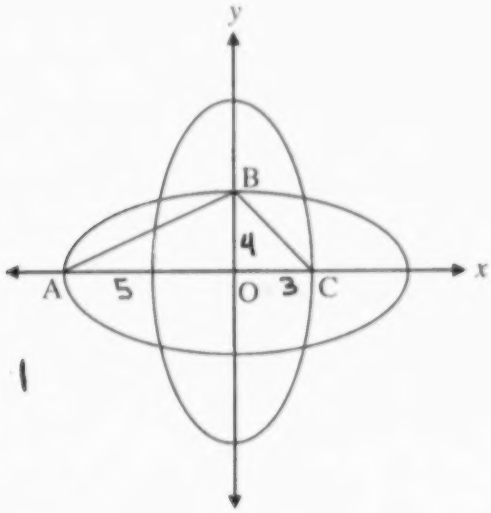
- base and height may be indicated on the diagram

# EXEMPLARS

10. The sketches of the ellipses with equations $\frac{x^2}{9}+\frac{y^2}{16}=1$ and $\frac{x^2}{25}+\frac{y^2}{4}=1$ are given below.

(The ellipses are not necessarily drawn to scale.)

Find the area of $\Delta ABC$.




$5+3=8=$ base

$4=$ height

$\frac{8\cdot 4}{2}=16u^2$

$\frac{x^2}{9}+\frac{y^2}{16}=1$

$\frac{x^2}{25}+\frac{y^2}{4}=1$

Horiz

$\sqrt{25}=5=x$ units

$\sqrt{4}=2=y$ units

$\sqrt{9}=3=x$ units

$\sqrt{16}=4=y$ units

vert

**Mark:** 2 out of 3

**Note(s):** • deduct 1 mark for incorrect height

# Question No. 11

**Outcome: F2, F3**

## Question

11. A circle has the equation $x^2 + y^2 + 6x - 10y + k = 0$.
Find the value of $k$ if the radius is 5.

3 marks

## Solution

### Method 1

$$x^2 + 6x + 9 + y^2 - 10y + 25 + k = 9 + 25$$
$$(x+3)^2 + (y-5)^2 = 9 + 25 - k$$

2 marks { 1 mark for completing square on $x$; 1 mark for completing square on $y$ }

$$\therefore r^2 = 34 - k$$
$$5^2 = 34 - k$$
$$25 = 34 - k$$

½ mark for substitution

$$k = 9$$

½ mark for final answer

### Method 2

$$r = \sqrt{\left(\frac{D}{2}\right)^2 + \left(\frac{E}{2}\right)^2 - k}$$

1 mark for formula

$$5 = \sqrt{\left(\frac{6}{2}\right)^2 + \left(\frac{-10}{2}\right)^2 - k}$$

1 mark for substitution of D & E

$$25 = 9 + 25 - k$$
$$k = 9$$

1 mark for solving for $k$

## Note(s)

- **deduct** 1 mark for $r = 34 - k$

# EXEMPLARS

11. A circle has the equation $x^2 + y^2 + 6x - 10y + k = 0$.
Find the value of $k$ if the radius is 5.

$$(x^2 + 6x + 9) + (y^2 - 10y + 25) = 9 + 25$$

$$(x+3)^2 + (y-5)^2 = 34$$

$$34 - 5 = 29$$

$$29 = k$$

**Mark:** **2 out of 3**
**Note(s):** • deduct 1 mark for using 5 instead of 25

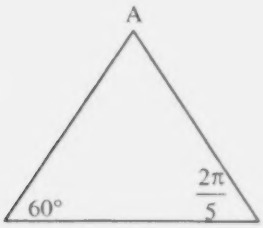
# Part 2—Restricted-Response Questions

## Question No. 27

**Outcome: A1**

### Question

27. Find the measure of $\angle A$. Your answer may be stated in radians or degrees.




1 mark

### Answer

**Method 1**

$$\frac{2\pi}{5} \cdot \frac{180°}{\pi} = \frac{2}{5}(180°) = 72°$$

$$\angle A = 180° - (60 + 72)° = 48°$$

**Method 2**

$$60° = \frac{\pi}{3}$$

$$\angle A = \pi - \left(\frac{\pi}{3} + \frac{2\pi}{5}\right)$$

$$= \frac{4\pi}{15}$$

### Note(s)

- give ½ mark for correct conversion

# Question No. 28

**Outcome: E3**

## Question

28. There are 8 points placed on a circle. Triangles are formed by using 3 of these points as vertices. How many different triangles are determined by these 8 points? (Leave your answer in factorial form.)

1 mark

## Answer

${}_8C_3$

$= \dfrac{8!}{5!3!}$

## Note(s)

- ½ mark ${}_8C_3$ as final answer
- ½ mark for ${}_8C_3 = 56$ (omitting factorial expression)

# Question No. 29

**Outcome: A3**

## Question

29. Find the numerical value of:

$$\sec\frac{\pi}{3}$$

1 mark

## Answer

2

## Note(s)

- give ½ mark if $\cos\frac{\pi}{3} = \frac{1}{2}$ is written

# Question No. 30

**Outcome: D4**

## Question

30. If $f(x) = \log_{16} x$, find the value of $f(4)$.

1 mark

## Answer

**Method 1**

$$f(4) = \log_{16} 4$$
$$= \frac{1}{2}$$

**Method 2**

$$y = \log_{16} 4$$
$$16^y = 4$$
$$\left(4^2\right)^y = 4$$
$$4^{2y} = 4$$
$$4^{2y} = 4^1$$
$$2y = 1$$
$$y = \frac{1}{2}$$

$$\therefore f(4) = \frac{1}{2}$$

## Note(s)

- give ½ mark for $16^y = 4$

# Question No. 31

**Outcome: D7**

## Question

31. If $x = e^{2\ln 3}$ then find the value of $x$ as a whole number.

1 mark

## Answer

### Method 1

$x = e^{2\ln 3} = \left(e^{\ln 3}\right)^2$

$x = 3^2$

$x = 9$

### Method 2

$$\begin{aligned} x &= e^{2\ln 3} \\ \ln e^{2\ln 3} &= \ln x \\ 2\ln 3(\ln e) &= \ln x \\ 2\ln 3 &= \ln x \\ \ln 3^2 &= \ln x \\ 9 &= x \end{aligned}$$

### Method 3

$x = e^{2\ln 3} = e^{\ln 3^2}$

$x = 3^2$

$x = 9$

## Note(s)

- give ½ mark for $3^2$ as a final answer

# Question No. 32

**Outcome: E4**

## Question

32. The binomial $(x+y)^9$ is expanded using the Binomial Theorem.

What is the value of $a$ in the term ${}_9C_6x^a y^6$?

1 mark

## Answer

$a = 9 - 6 = 3$

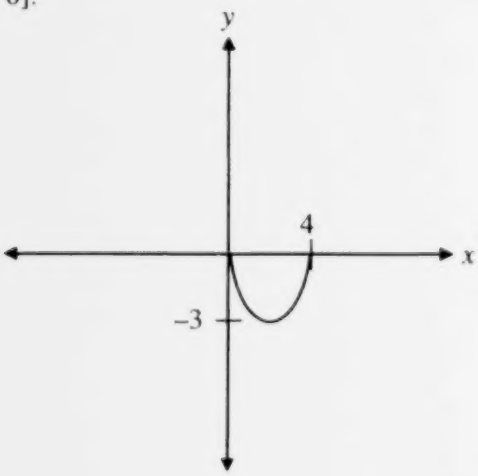
# Question No. 33

**Outcome: B3**

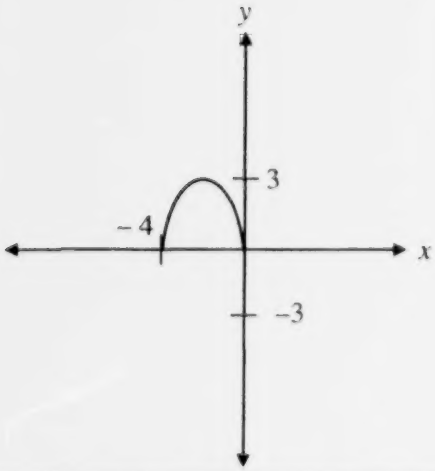
## Question

33. The sketch of $f(x)$, in the interval $[0, 4]$ is given below. If $f(x)$ is an odd function, sketch $f(x)$ in the interval $[-4, 0]$.

1 mark




## Answer




## Note(s)

- **deduct** ½ mark for no units on axes or minor inaccuracies in graph

# Question No. 34

**Outcome: E1**

## Question

34. Find the number of permutations of all the letters of the word PEPPER. State your answer as a whole number.

1 mark

## Answer

$$\frac{6!}{3!2!} = 60$$

## Note(s)

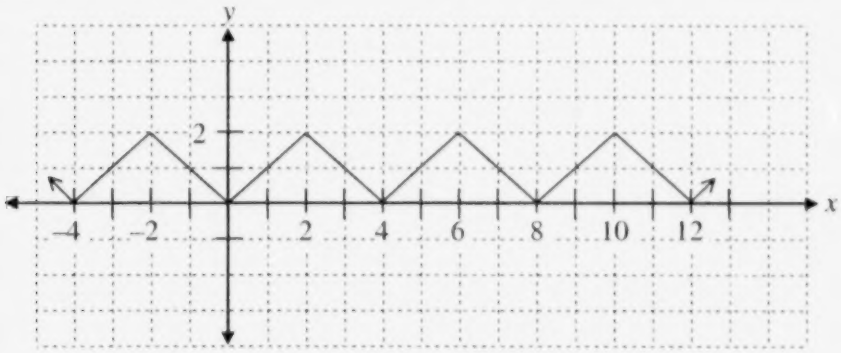
- give ½ mark for $\frac{6!}{3!2!}$

# Question No. 35

**Outcome: A6, B7**

## Question

35. The graph of the periodic function $f(x)$ is given below.




The value of $f(98)$ is: _____

1 mark

## Answer

$f(98) = 2$

## Note(s)

- give ½ mark for incorrect answer but correctly indicating that the period is 4.

# Question No. 36

**Outcome: G2**

## Question

36. The probabilities of the independent events A and B are P(A) = 0.5 and P(B) = 0.6. Find the value of P(A or B).

1 mark

## Answer

$$\begin{aligned} P(\text{A or B}) &= P(A)+P(B)-P(\text{A and B}) \\ &= 0.5+0.6-(0.5)(0.6) \\ &= 0.5+0.6-0.3 \\ &= 0.8 \end{aligned}$$

## Note(s)

- give 0 marks for $P(\text{A or B}) = 0.5+0.6$
  $= 1.1$

# Question No. 37

**Outcome: G3**

## Question

37. The probability that Barry passes History is $\frac{3}{4}$. The probability that he passes Science is $\frac{4}{5}$.

Find the probability that Barry passes History but fails Science.

1 mark

## Answer

$$\frac{3}{4}\left(1-\frac{4}{5}\right)=\frac{3}{4}\left(\frac{1}{5}\right)=\frac{3}{20}$$

## Note(s)

- give ½ mark for $\frac{3}{4}\times\frac{4}{5}$ OR $\frac{1}{4}\times\frac{1}{5}$
- give ½ mark for P(Fails Science) = $\frac{1}{5}$

# Question No. 38

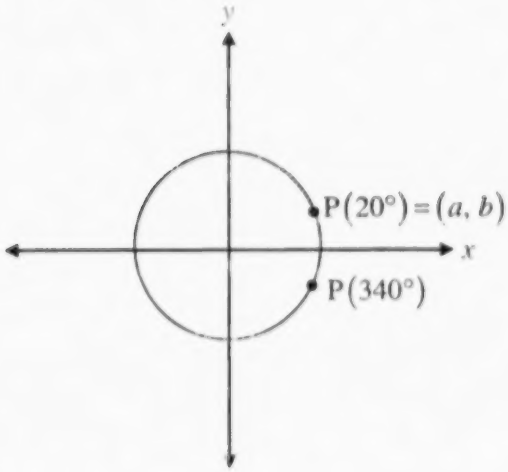
**Outcome: A2**

## Question

38. $P(\theta)$ is the terminal point on the unit circle. The coordinates of $P(20°)$ are $(a, b)$.
What are the coordinates of $P(340°)$ in terms of $a$ and $b$?

1 mark

## Answer




$340° = 360° - 20°$

$\therefore$ coordinates of $P(340°)$ are $(a, -b)$

## Note(s)

- no part marks

# Question No. 39

**Outcome: H1**

## Question

39. Find an expression for the 50th term of the geometric sequence:
$2^4, 2^6, 2^8, \ldots$

1 mark

## Answer

### Method 1

$n^{th}$ term $= 2^{2n+2}$

$50^{th}$ term $= 2^{2(50)+2}$

$= 2^{102}$

OR

$4\left(2^{2n}\right)$

$= 4\left(2^{2(50)}\right)$

$= 4\left(2^{100}\right)$

OR

$t_{50} = 16\,(4)^{49}$

### Method 2

$r = \dfrac{2^6}{2^4} = 2^2$

$\therefore t_{50} = 2^4\left(2^2\right)^{49}$

$= 2^4 2^{98}$

$= 2^{102}$

1 mark for any of these forms

## Note(s)

- give 1 mark for any equivalent answer
- the expression does not need to be simplified
- award ½ mark for correct value of $r$

# Question No. 40

**Outcome: B1, B2**

## Question

40. The $x$-intercept of $y = f(x)$ is 5.
    Find the $x$-intercept of $y = 2f(x-3)$.

1 mark

## Answer

$5 + 3 = 8$

## Note(s)

- ½ mark for stating (8, 0)

# Question No. 41

**Outcome: B3**

## Question

41. Find the range of the function:
$f(x) = \cos x + \cos(-x)$

1 mark

## Answer

$$\begin{aligned} f(x) &= \cos x + \cos(-x) \\ &= \cos x + \cos x \\ &= 2\cos x \end{aligned}$$

$\therefore$ Range is $[-2, 2]$

## Note(s)

- no mark for $f(x) = 0$

  $\therefore range = \{0\}$
- give ½ mark for $f(x) = 2\cos x$

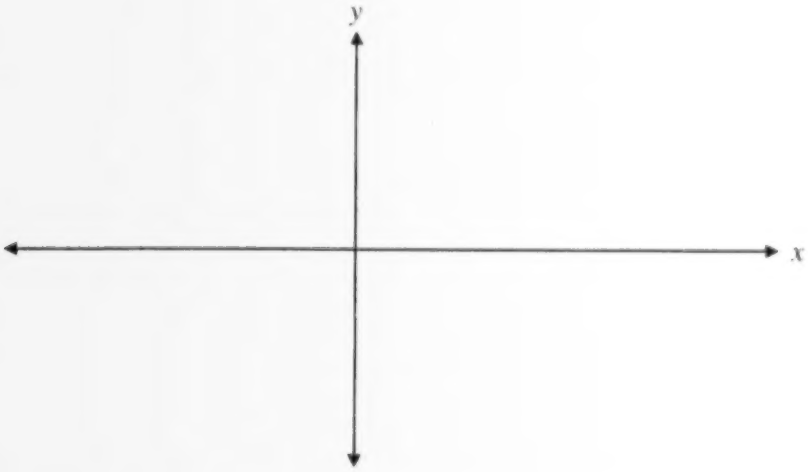
# Part 2—Open-Response Questions

## Question No. 42

**Outcome: A6**

### Question

42. a) Sketch a clearly labelled graph of at least one period of the curve $y = \cos x$. **1 mark**

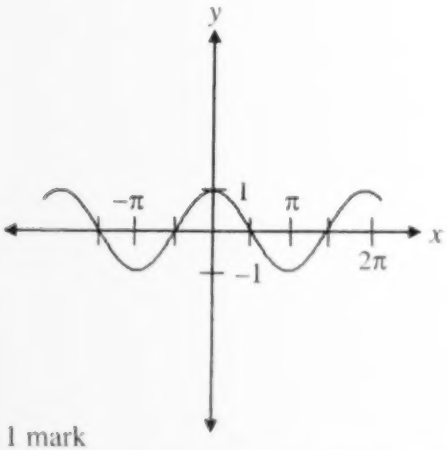



b) State the $y$-intercept of $y = \cos x$.

$y$-intercept = _____ **1 mark**

### Solution

a)




1 mark

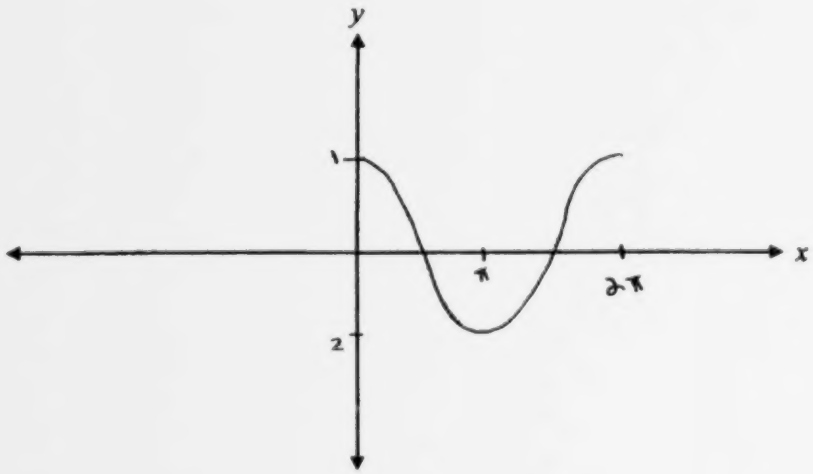
b) $y$-intercept = 1 1 mark

### Note(s)

- **deduct** ½ mark for any missing units
- no part marks in b), answer in b) is independent of a); do not mark with consistency of a)

# EXEMPLARS

42. a) Sketch a clearly labelled graph of at least one period of the curve $y = \cos x$.




| | |
|---|---|
| **Mark:** | **½ out of 1** |
| **Note(s):** | • deduct ½ mark for incorrect unit on $y$-axis |

b) State the $y$-intercept of $y = \cos x$.

$y$-intercept = 1

| |
|---|
| **Mark: 1 out of 1** |

# Question No. 43

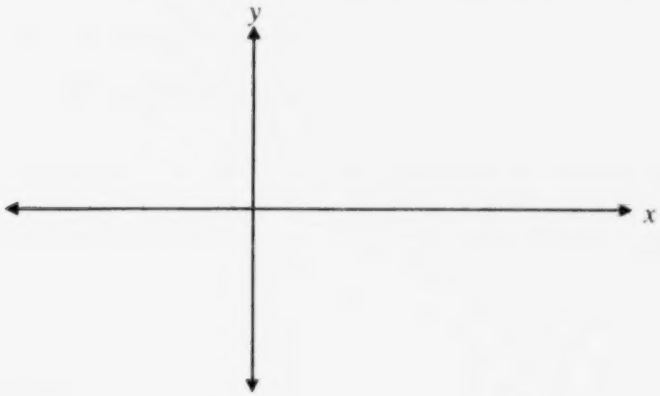
**Outcome: B5**

## Question

43. a) Find the $x$-intercept(s) of $f(x)=|x^3-8|$. **1 mark**

$x$-intercept(s) = _____

b) Sketch a clearly labelled graph of $f(x)=|x^3-8|$. **2 marks**




## Solution

a) $x$-intercept(s)

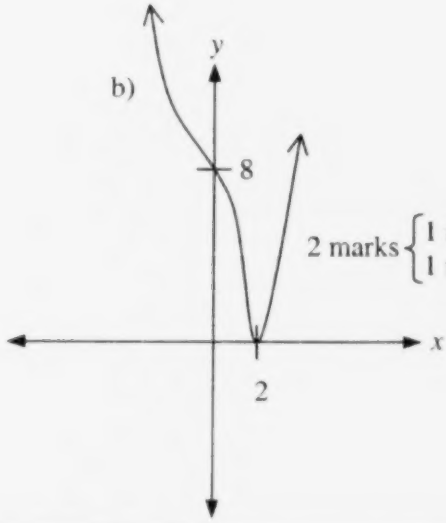
$0=|x^3-8|$ 1 mark

$0=x^3-8$

$8=x^3$

$x-\text{intercept}=2$

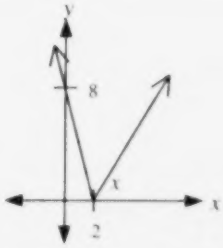
b)

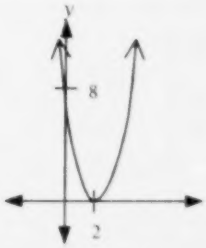



2 marks { 1 mark for horizontal translation; 1 mark for absolute value }

## Note(s)

- part a) ½ mark if $x$-intercept is written as (2, 0)
- maximum of 1 mark in part (b) for either of these graphs
- deduct 1 mark if the graph is not a cubic shape (inflection at (0, 8))



# EXEMPLARS

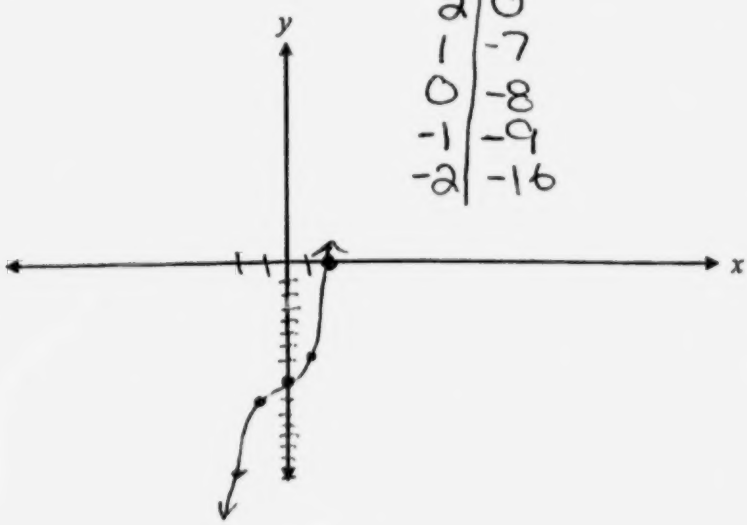
43. a) Find the $x$-intercept(s) of $f(x) = |x^3 - 8|$.

$x$-intercept(s) = 2

**Mark: 1 out of 1**

b) Sketch a clearly labelled graph of $f(x) = |x^3 - 8|$.

| $x$ | $y$ |
|---|---|
| 2 | 0 |
| 1 | -7 |
| 0 | -8 |
| -1 | -9 |
| -2 | -16 |




**Mark: 1 out of 2**
**Note(s):** • deduct 1 mark for ignoring absolute value

# Question No. 44

**Outcome: D6**

## Question

44. Solve for $x$:

4 marks

$$\log_2(2-x) = 1 - \log_2(3-x)$$

## Solution

### Method 1

$$\log_2(2-x) + \log_2(3-x) = 1$$

$$\log_2\left[(2-x)(3-x)\right] = 1$$

$$\log_2\left(6-5x+x^2\right) = 1$$ 1 mark for log theorem

$$6-5x+x^2 = 2^1$$ 1 mark for exponential form

$$x^2-5x+4=0$$

$$(x-4)(x-1)=0$$

$$\cancel{x=4} \quad x=1$$ 1 mark for solving for $x$

$$\therefore x=1$$ 1 mark for discarding the extraneous root

### Method 2

$$\log_2(2-x) = \log_2 2 - \log(3-x)$$ ½ mark for $1 = \log_2 2$

$$\log_2(2-x) = \log_2\left[\frac{2}{(3-x)}\right]$$ 1 mark for log theorem

$$(2-x) = \frac{2}{(3-x)}$$ ½ mark

$$(2-x)(3-x) = 2$$

$$6-5x+x^2 = 2$$

$$x^2-5x+x^2 = 2$$

$$x^2-5x+4=0$$

$$(x-4)(x-1)=0$$

$$\cancel{x=4} \quad x=1$$ 1 mark for solving for $x$

$$\therefore x=1$$ 1 mark for discarding the extraneous root

# EXEMPLARS

44. Solve for $x$:

$$\log_2(2-x) = 1 - \log_2(3-x)$$

$$\log_2(2-x) + \log_2(3-x) = 1$$

$$\log_2[(2-x)(3-x)] = 1$$

$$\log_2(6-5x+x^2) = 1$$

$$2^1 = x^2 - 5x + 6$$

$$0 = x^2 - 5x + 8$$

$$0 = (x- \quad)(x- \quad)$$

**Mark:** **2 out of 4**

**Note(s):** • give 2 marks for log theorem and correct change to exponential form

# Question No. 45

**Outcome: A4, C1**

## Question

45. Solve for $x$ in the interval $[0, 2\pi]$:

$$\cos^2 x = \sin^2 x + \sin x$$

4 marks

## Solution

| | |
|---|---|
| $1 - \sin^2 x = \sin^2 x + \sin x$ | 1 mark |
| $0 = 2\sin^2 x + \sin x - 1$ | |
| $0 = (2\sin x - 1)(\sin x + 1)$ | 1 mark |
| $\sin x = \frac{1}{2} \qquad \sin x = -1$ | ½ mark |
| $x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2}$ | 1½ marks (½ mark each) |

## Note(s)

- **deduct** ½ mark if an equation is changed to an expression: $(2\sin x - 1)(\sin x + 1)$; i.e., "= 0" has been dropped
- maximum of 3 marks if final answers are in degrees $x = 30°, 150°, 270°$

# EXEMPLARS

45. Solve for $x$ in the interval $[0, 2\pi]$:

$$\cos^2 x = \sin^2 x + \sin x$$

$1 - \sin^2 x = \sin^2 x + \sin x$

$0 = 2\sin^2 x + \sin x$

$0 = \sin x(2\sin x + 1)$

$x = 0, -\frac{1}{2}$

$x = 0, \pi, 2\pi, -\frac{\pi}{6}, -\frac{7\pi}{6}$

| | |
|---|---|
| **Mark:** | **1 out of 4** |
| **Note(s):** | • 1 mark for changing $\cos^2 x$ into $1 - \sin^2 x$<br>• deduct 3 marks for numerous errors after 1st line |

# Question No. 46

**Outcome: C2, A3**

## Question

46. a) Prove the identity $\tan x = \dfrac{\sin 2x}{1+\cos 2x}$. **3 marks**

b) Use the identity $\tan x = \dfrac{\sin 2x}{1+\cos 2x}$ to find an exact value of $\tan\left(\dfrac{\pi}{8}\right)$, in simplified form. **2 marks**

## Solution

a)

$$\text{RHS} = \frac{\sin 2x}{1+\cos 2x}$$

$$= \frac{2\sin x\cos x}{1+\left(2\cos^2 x-1\right)}$$ 1 mark (½ mark per identity)

$$= \frac{2\sin x\cos x}{2\cos^2 x}$$ 1 mark for simplifying

$$= \frac{\sin x}{\cos x}$$ 1 mark for simplifying

$$= \tan x = \text{LHS}$$

b) **Method 1**

$$\tan\left(\frac{\pi}{8}\right) = \frac{\sin 2\left(\frac{\pi}{8}\right)}{1+\cos 2\left(\frac{\pi}{8}\right)}$$

$$= \frac{\sin\frac{\pi}{4}}{1+\cos\frac{\pi}{4}}$$ 1 mark

$$= \frac{\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}$$ ½ mark

$$= \frac{1}{\sqrt{2}+1}$$ ½ mark

**Method 2**

$$\tan\left(\frac{\pi}{8}\right) = \frac{\sin 2\left(\frac{\pi}{8}\right)}{1+\cos 2\left(\frac{\pi}{8}\right)}$$

$$= \frac{\sin\frac{\pi}{4}}{1+\cos\frac{\pi}{4}}$$ 1 mark

$$= \frac{\frac{\sqrt{2}}{2}}{1+\frac{\sqrt{2}}{2}}$$ ½ mark

$$= \frac{\sqrt{2}}{2+\sqrt{2}}$$ ½ mark

**Note(s)**

- equivalent answer for part b): $\sqrt{2}-1$

# EXEMPLARS

46. a) Prove the identity $\tan x = \dfrac{\sin 2x}{1+\cos 2x}$.

$$\tan x = \frac{2\sin x\cos x}{1+\cos^2 x+\sin^2 x}$$

$$\tan x = \frac{2\sin x\cos x}{1+1-\sin^2 x-\sin^2 x}$$

$$\frac{\sin x}{\cos x} = \frac{2\sin x\cos x}{2-2\sin^2 x}$$

$$2(1-\sin^2 x)$$

$$\frac{\cancel{2}\sin x\cancel{\cos x}}{\cancel{2}(\cos^{\cancel{2}} x)}$$

$$\frac{\sin x}{\cos x} = \frac{\sin x}{\cos x}$$

**Mark:** **2 out of 3**
**Note(s):** • deduct 1 mark for bad form (writing as an equation and losing numerator)

b) Use the identity $\tan x = \dfrac{\sin 2x}{\quad}$ to find an exact value of $\tan\left(\dfrac{\pi}{8}\right)$, in simplified form.

$$= \frac{\sin 2\frac{\pi}{8}}{1+\cos 2(\frac{\pi}{8})}$$

$$= \frac{\sin \pi/4}{1+\cos \pi/4}$$

$$= \frac{\sqrt{2}/2}{1+\sqrt{2}/2}$$

$$= \frac{\sqrt{2}/2}{2\sqrt{2}/2}$$

$$= \frac{2\sqrt{2}}{4\sqrt{2}}$$

$$= 1/2$$

$= 1/2$

**Mark:** **1½ out of 2**
**Note(s):** • deduct ½ mark for incorrect simplification

# Question No. 47

**Outcome: A2, C2**

## Question

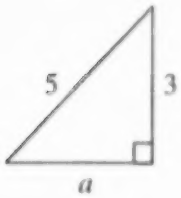
47. The angle $\alpha$ is in quadrant II and $\sin\alpha = \frac{3}{5}$.

a) Find the exact numerical value of $\cos\alpha$. — 1 mark

b) Find the exact numerical value of $\cos(\alpha - 60°)$. — 3 marks

## Solution

a) since $\sin\alpha = \frac{3}{5}$ it follows that




$$\left.\begin{aligned} &5^2 = 3^2 + a^2 \\ &\pm 4 = a \\ &\cos\alpha = -\frac{4}{5} \text{ since } \alpha \in \text{QII} \end{aligned}\right\} \text{ 1 mark}$$

b) $\cos(\alpha - 60°) = \cos\alpha\cos 60° + \sin\alpha\sin 60°$ — ½ mark

$$\therefore \cos(\alpha - 60°) = -\frac{4}{5}\left(\frac{1}{2}\right) + \frac{3}{5}\left(\frac{\sqrt{3}}{2}\right) \rightarrow$$

1½ mark (½ mark for each of the following function values : $-\frac{4}{5}$, $\frac{1}{2}$ and $\frac{\sqrt{3}}{2}$)

$$= \frac{3\sqrt{3} - 4}{10}$$

1 mark

## Note(s)

- **deduct** ½ mark for $\cos\alpha = \frac{4}{5}$
- mark part b) consistent with part a)

# EXEMPLARS

47. The angle $\alpha$ is in quadrant II and $\sin\alpha = \frac{3}{5}$.

a) Find the exact numerical value of $\cos\alpha$.

$\sin x = \frac{3}{5} \frac{y}{r}$

$x^2 + 3^2 = 5^2$

$x = 4$

$\cos x = \frac{x}{r} = -\frac{4}{5}$

| |
|---|
| **Mark: 1 out of 1** |

b) Find the exact numerical value of $\cos(\alpha - 60°)$.

$\cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos x \cos 60° + \sin x \sin 60°$

$-\frac{4}{5}\cos 60° + \frac{3}{5}\sin 60°$

$-\frac{4}{5}\left(\frac{-1}{2}\right) + \frac{3}{5}\left(\frac{1}{2}\right)$

$= \frac{4}{10} + \frac{3}{10}$

$= \frac{7}{10}$

| |
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| **Mark: 2 out of 3**<br>**Note(s):** • deduct ½ for each incorrect function value i.e. –½ and ½<br>• no marks deducted for consistently substituting $x$ for $\alpha$ |

# Question No. 48

**Outcome: G2**

## Question

48. A bag contains 4 blue marbles and 6 red marbles.
A marble is selected at random from this bag and is replaced with a marble of the other colour.
A second marble is selected at random from the new mixture of 10 marbles in this bag.
Find the probability that the second marble selected is red.

4 marks

## Solution

**Method 1**

$$\left.\begin{aligned} P(R_2) &= P\left[(R_1 \text{ and } R_2) \text{ OR } (B_1 \text{ and } R_2)\right] \\ &= P(R_2|R_1) + P(R_2|B_1) \end{aligned}\right\} \text{ 1 mark for method}$$

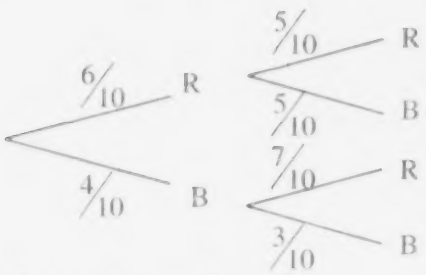
$$P(R_2) = \left(\frac{6}{10}\right)\left(\frac{5}{10}\right) + \left(\frac{4}{10}\right)\left(\frac{7}{10}\right) \qquad \text{2 marks (½ for each fraction)}$$

$$= \frac{30}{100} + \frac{28}{100} = \frac{58}{100} \text{ OR } \frac{29}{50} \text{ OR } 58\% \qquad \text{1 mark}$$

## Note(s)

- full marks if work begins on line 3

**Method 2**




$$\left(\frac{6}{10}\right)\left(\frac{5}{10}\right) = \frac{30}{100} \qquad \text{1 mark}$$

$$\left(\frac{4}{10}\right)\left(\frac{7}{10}\right) = \frac{28}{100} \qquad \text{1 mark}$$

1 mark for correct tree diagram

$$P(\text{Second Red}) = \frac{30}{100} + \frac{28}{100} = \frac{58}{100} \text{ OR } \frac{29}{50} \text{ OR } 58\% \qquad \text{1 mark}$$

# EXEMPLARS

48. A bag contains 4 blue marbles and 6 red marbles.
A marble is selected at random from this bag and is replaced with a marble of the other colour.
A second marble is selected at random from the new mixture of 10 marbles in this bag.
Find the probability that the second marble selected is red.

$\frac{1}{2}$ 3B, 7R → 7/10

$\frac{1}{2}$ 5B, 5R → 5/10

$$P(R) = \left(\frac{1}{2}\right)\left(\frac{5}{10}\right) + \left(\frac{1}{2}\right)\left(\frac{7}{10}\right)$$

$$= \left(\frac{1}{2}\right)\left(\frac{5}{10} + \frac{7}{10}\right)$$

$$= \frac{6}{10} = \boxed{\frac{3}{5}}$$

**Mark:** **3 out of 4**
**Note(s):** • deduct ½ mark each for 2 incorrect fractions

# Question No. 49

**Outcome: E1**

## Question

49. Solve for $n$:

$$\frac{(n+4)!}{(n+2)!} = 2$$

2 marks

## Solution

### Method 1

$$\frac{(n+4)(n+3)(n+2)!}{(n+2)!} = 2$$ ½ mark

$n^2 + 7n + 12 = 2$

$n^2 + 7n + 10 = 0$

$(n+5)(n+2) = 0$

$n = -5 \qquad n = -2$

1 mark

$n = -2$ ½ mark for final answer

### Method 2

$$\frac{(n+4)(n+3)(n+2)!}{(n+2)!} = 2$$ ½ mark

$(n+4)(n+3) = 2$ ½ mark

If the product of two consecutive positive integers is 2, then the greater integer ($n$ + 4) equals 2 and the smaller integer ($n$ + 3) equals 1.

$n + 4 = 2$ ½ mark

$n = -2$ ½ mark

# EXEMPLARS

49. Solve for $n$:

$$\frac{(n+4)!}{(n+2)!} = 2$$

$$\frac{(n+4)(n+3)(n+2)!}{(n+2)!} = 2$$

$$n^2 + 7n + 12 = 2$$

$$n^2 + 7n + 10 = 0$$

$$(n+5)(n+2) = 0$$

$$n = \cancel{-5} \quad n = \cancel{-2}$$

no solution, since n cannot be negative

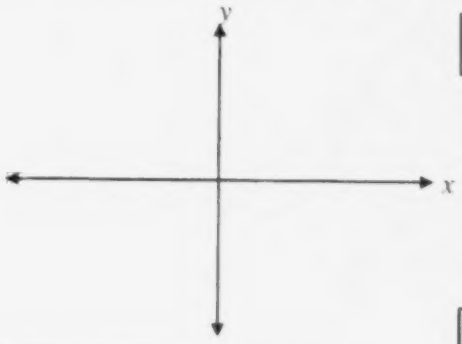
| | |
|---|---|
| **Mark:** | **1½ out of 2** |
| **Note(s):** | • deduct ½ mark for rejecting $(-2)$ |

# Question No. 50

**Outcome: B7**

## Question

50. a) Sketch a clearly labelled graph of at least one period of $f(x) = \sin(\pi x) + 2$.

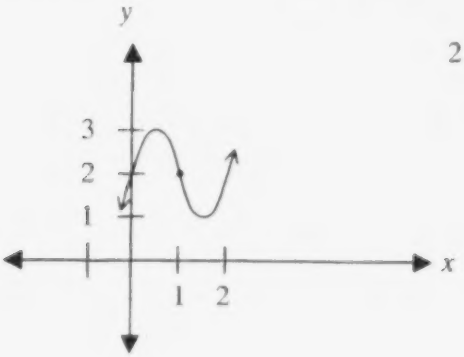



2 marks

b) The period of $f(x) = \sin(\pi x) + 2$ is _____ 1 mark

c) The range of $f(x) = \sin(\pi x) + 2$ is _____ 1 mark

## Solution




2 marks $\begin{cases} \text{1 mark for vertical translation} \\ \text{1 mark for horizontal compression} \end{cases}$

Period $= \dfrac{2\pi}{\pi} = 2$ 1 mark

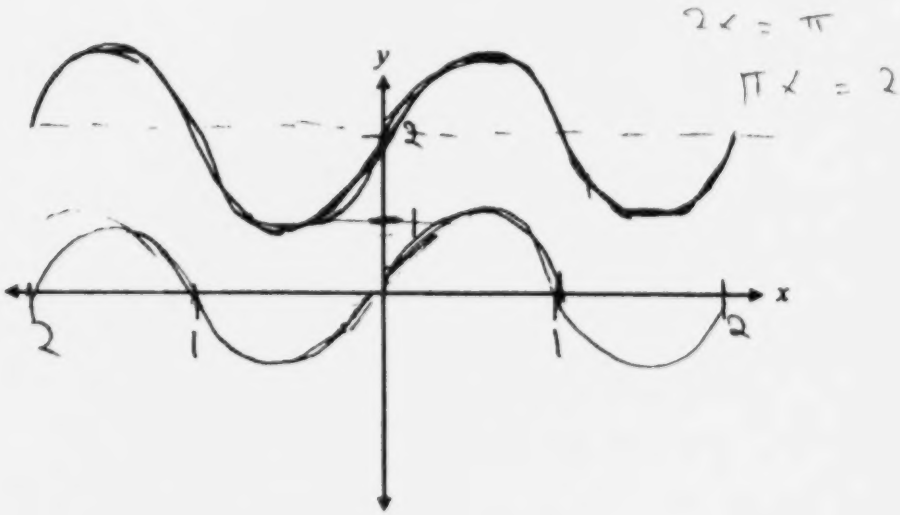
Range $= [1, 3]$ 1 mark

## Note(s)

- arrows are not required in the graph
- ½ mark for $[1, 3)$ OR $(1, 3]$ OR $(1, 3)$ OR $1 < y \le 3$ OR $1 \le y < 3$ OR $1 < y < 3$

# EXEMPLARS

50. a) Sketch a clearly labelled graph of at least one period of $f(x) = \sin(\pi x) + 2$.




| | |
|---|---|
| **Mark:** | **1 out of 2** |
| **Note(s):** | • ambiguous final answer |

b) The period of $f(x) = \sin(\pi x) + 2$ 2

| |
|---|
| **Mark: 1 out of 1** |

c) The range of $f(x) = \sin(\pi x) + 2$ [3, 1]

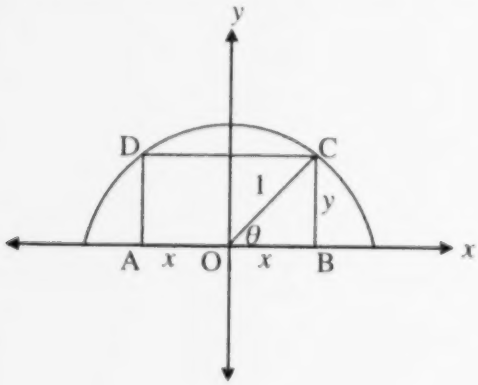
| |
|---|
| **Mark: 0 out of 1** |

# Question No. 51

**Outcome: C2**

## Question

51. C and D are points on the unit circle and A and B are on the $x$-axis.
If $\angle COB = \theta$, prove that the area of rectangle $ABCD = \sin 2\theta$.




3 marks

## Solution

$AO = OB = x$ and $CB = y$

Area $= 2xy$ } 1 mark

But $\sin\theta = \frac{y}{1}$ and $\cos\theta = \frac{x}{1}$ 1 mark

(or coordinates of C are $(\cos\theta, \sin\theta) = (x, y)$)

$\therefore$ Area $= 2xy$

$2\sin\theta\cos\theta$

$= \sin 2\theta$ } 1 mark for identity

## Note(s)

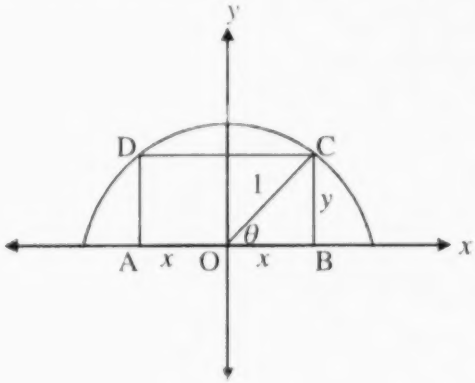
- give full marks for Area $= 2xy$

$= 2\sin\theta\cos\theta$

$= \sin 2\theta$

# EXEMPLARS

51. C and D are points on the unit circle and A and B are on the $x$-axis.
If $\angle COB = \theta$, prove that the area of rectangle $ABCD = \sin 2\theta$.




$\sin 2\theta = 2\sin\theta\cos\theta$

$A = b \times h$.

$\sin\theta = y$
$\cos\theta = x$

base $= 2\cos x$
height $= \sin y$

$\therefore A = b \times h$
$= (2\cos x)(\sin y)$
$= 2\sin\theta$

**Mark:** **1 out of 3**
**Note(s):** • give 1 mark for $\sin\theta = y$ and $\cos\theta = x$